EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 20th April 2016 09.30 to 12.30

Module 3A3

FLUID MECHANICS II

Answer not more than ***five*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Compressible Flow Data Book (38 pages)
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Show that the Mach number behind a strong shock wave has a maximum value given by

$$M_s^2 = \frac{\gamma - 1}{2\gamma} \quad . \qquad [20\%]$$

(b) A pipe contains air flowing at a constant velocity of 250 ms^{-1} with a constant temperature of 500 K. The pipe diameter is constant and there is no friction or heat transfer. At a certain point in the pipe the pressure is measured as 150 kPa. At the same point there is a fast-acting valve. Suddenly the valve is closed.

(i) What is the speed of the resulting shock wave relative to the pipe? [60%]
(ii) Calculate the pressure acting on the upstream side of the closed valve. [20%]

2 Air flows from a reservoir through a frictionless convergent-divergent nozzle into a pipe. There is friction in the pipe and the pipe exit is choked.

(a) Draw a T-s diagram to illustrate the flow from reservoir to pipe exit, for the case where the flow at the nozzle exit is:

(i) subsonic; [15%]
(ii) supersonic. [15%]

(b) Starting from the case where the nozzle exit is supersonic and the pipe exit is choked, the pipe is made longer by adding extra sections with the same diameter. Draw another T-s diagram to illustrate the changes in the flow pattern. Sketch the possible flow scenarios. [30%]

(c) When the length of the pipe is 5 m, the Mach number at the nozzle exit is found to be M = 1.5 and the pipe exit is choked. A shock wave is found to occur in the pipe with a strength corresponding to a Mach number of 1.36. The skin friction coefficient is 0.0025. Find the diameter of the pipe and the location of the shock wave. [40%]

3 (a) Show that the Riemann invariant across a right-running infinitesimal sound wave in a perfect gas is

$$V - \frac{2a}{\gamma - 1} = \text{constant} \quad .$$ [30%]

(b) Fig. 1 shows the *x*-*t* diagram for a piston moving to the left in a cylinder of constant diameter filled with air. At time $t = 0$ the piston is located at $x = 0$. Initially, the air to the right of the piston has zero velocity, a pressure of 200 kPa and a temperature of 400 K. The piston accelerates to the left, reaching a constant velocity at the point A.

(i) The density in region 2 is found to be 60% of that in region 1. Calculate the velocity of the piston along the path AB, and the pressure and temperature in region 2. [40%]

(ii) If, in a different scenario, the acceleration of the piston is such that its velocity at the point A is equal to the local speed of sound, calculate the pressure and temperature in region 2. [30%]

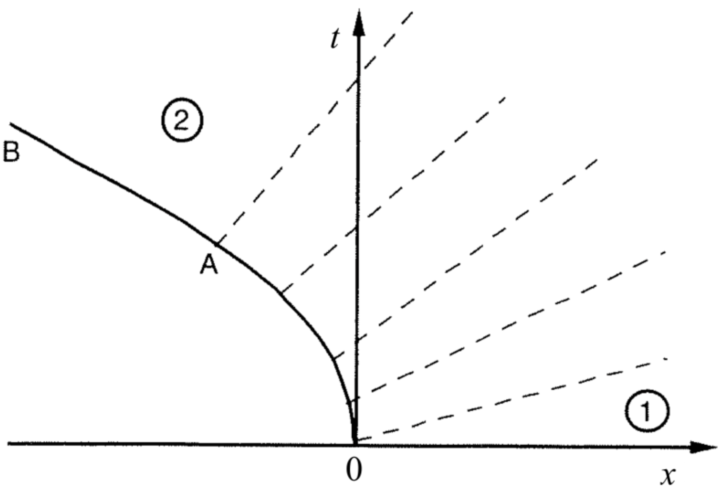

Fig. 1

4 Consider an oblique shock wave in a supersonic flow in dry air.

(a) Express the density ratio across the shock in terms only of the ratio of specific heats γ for the case where the upstream Mach number tends to infinity. [20%]

(b) Show that the deflection angle θ and the shock angle β may be related by:

$$\tan\theta = \frac{\sin 2\beta}{\gamma + \cos 2\beta} \quad .$$ [25%]

(c) Calculate the maximum possible deflection angle θ_{max} for this case. As the upstream Mach number is reduced to attainable flight Mach numbers, do you expect θ_{max} to increase or decrease? [30%]

(d) The nose of a supersonic aircraft is shown in simplified form in Fig. 2. For each of the following cases, carefully sketch and label the flow features when the flight Mach number M is such that:

(i) $\theta < \theta_{\text{max}}$

(ii) $\theta > \theta_{\text{max}}$ [25%]

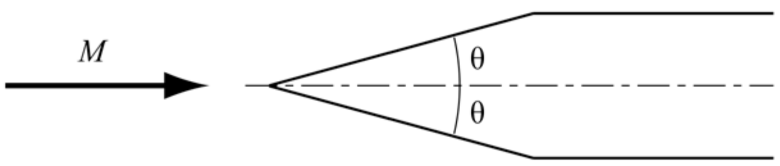

Fig. 2.

5 A supersonic aircraft is optimised for flight at a Mach number of 2.00 at an altitude of 18,000 m. During development it is found that the aircraft will not meet its range requirement and it is proposed to mount a fuel tank on the flat underside of the aircraft. The tank, sketched in Fig. 3 is a two-dimensional extruded shape, 1 m wide (into the page), 1.5 m long and 30 cm deep with 18^o angle ramps at both the leading and trailing edges. You may treat the flow as two-dimensional.

(a) On a detailed sketch, carefully label all of the principal supersonic flow features in the symmetry plane. [30%]

(b) Calculate the increase in engine cruise thrust required to overcome the additional drag of the tank. You may neglect friction and boundary layer effects. [60%]

(c) It is found that the tank size needs to be increased by a further 10%. Which *one* linear tank dimension would you choose to increase: depth, width or length? Give brief reasons for your answer. [10%]

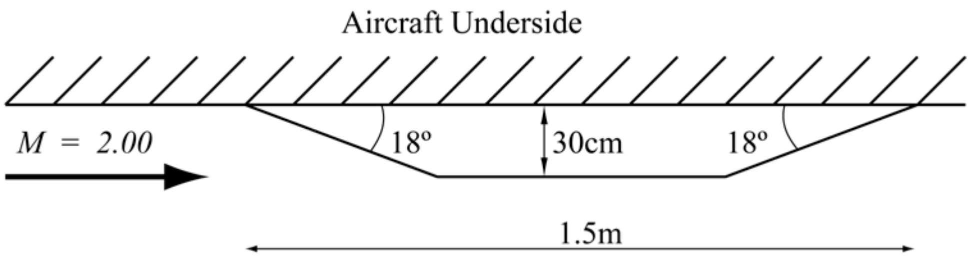

Fig. 3. (not to scale)

6 A one-dimensional disturbance u convecting in the $+x$ direction at speed A is described by the wave equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0 \quad .$$

In order to solve this numerically the following difference equation is derived

$$u_i^{n+1} = u_i^n - c\left(u_i^n - u_{i-1}^n\right) \quad .$$

where $t = n\ \Delta t$, distance $x = i\ \Delta x$ and $c = A\ \Delta t/\Delta x$.

(a) Determine the order of accuracy of this numerical scheme. [40%]

(b) By considering a sawtooth perturbation of small amplitude ε (the perturbation varies grid-point to grid-point from $+\varepsilon$ to $-\varepsilon$), determine the range of c for which the scheme is stable. [30%]

(c) The difference scheme is modified to

$$u_i^{n+1} = u_i^n - c\left(u_{i+1}^n - u_i^n\right) \quad .$$

Show that this scheme is unstable for all values of c . Why is this the case? [30%]

7 Note that part (a) of this question is not related to part (b).

(a) Partial differential equations can be classed as hyperbolic, elliptic or parabolic. Give a simple example of an equation from each class, describe the relevant initial and boundary conditions, and explain how its signal propagation behaviour can be exploited to devise a numerical solution method. [50%]

(b) (i) Derive the Euler work equation for turbomachinery stating clearly any assumptions. [25%]

(ii) Sketch an axial compressor blade row and the freestream and boundary layer wake velocity distribution at a downstream plane (the “stator plane”). Using velocity triangles and the Euler work equation show that the wake fluid is hotter than the free stream fluid in the downstream stator plane. [25%]

8 Consider two axial turbine stages, one with 50% reaction (the "reaction stage") and the other with 0% reaction (the "impulse stage"). Both operate with constant axial velocity V_x and density ρ. The mean-line flows are at constant radius.

(a) For each stage sketch the velocity triangles and derive the stage loading coefficients,

$$\psi = \Delta h_0 / U^2 \quad . \qquad [25\%]$$

(b) The loss coefficient for each blade row of the reaction stage is Y_R and for each blade row of the impulse stage, Y_I (for simplicity, stator and rotor coefficients are assumed equal). For each stage derive an overall total pressure loss coefficient in the form

$$\frac{\Delta p_0}{\frac{1}{2}\rho V_x^2} \quad .$$

in terms of the blade loss coefficients and the flow coefficient

$$\phi = \frac{V_x}{U} \quad . \qquad [50\%]$$

(c) Discuss the likely relative magnitudes of the two loss coefficients Y_R and Y_I in terms of the stage loading. For the same work output, two stages of the reaction design equal one stage of the impulse design. Which design is likely to be more efficient? Sketch how the "break-even" ratio of loss coefficients varies with flow coefficient. [25%]

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3A3 Fluid Mechanics II – 2016
Numerical answers

Q1
b) i) M_s=1.389, c_s=372.5m/s
ii) p_s=313.1kPa

Q2
c) D=0.296m, L_{12}=1.498m

Q3
b) i) V_2=194.4m/s, p_2=97.8kPa, T_2=326.7K ii) p_2=55.66kPa, T_2=277.5K

Q4
c) β=67.79°, θ=45.58°

Q5
b) D=5.041kN
c) increase length

Q6
a) First order accurate in time and space
c) $-1<1+2c<1$

HB 2015