Version TPH/3

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 26th April 2017 9.30 to 12.30

Module 3A3

FLUID MECHANICS II

*Answer not more than **five** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Compressible Flow Data Book
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A jet engine is mounted on a stationary test stand. The atmospheric pressure is P_a and the inlet velocity can be taken as negligible. At the exit, the static pressure is P_x , the jet velocity is V_x and the cross-sectional area is A_x .

(a) Show that the thrust T produced by the engine is given by

$$T = F_x - A_x P_a$$

where F_x is the impulse function at the exit. [20%]

(b) The engine is fitted with a convergent-divergent nozzle with a fixed throat diameter of 0.2 m. The stagnation pressure measured upstream of the throat is 3.67 bar and atmospheric pressure is 1 bar. The nozzle exit area is variable, and for the purposes of the test the nozzle exit area is set such that the flow is isentropic throughout. Calculate the nozzle exit area and the thrust produced by the engine. [30%]

(c) The engine test is repeated with the variable-area nozzle adjusted to give an exit to throat area ratio equal to 1.1. The test is then repeated again with an exit to throat area ratio equal to 1.3. The nozzle inlet stagnation pressure and the atmospheric pressure both remain unchanged. For each case sketch the flow pattern in the jet and calculate the thrust of the engine. Comment on the variation of thrust with nozzle exit area. [50%]

2 (a) Draw an *h*-*s* diagram to illustrate frictionless one-dimensional flow in a constant area duct with heat transfer. Label all points of significance, and indicate clearly the path taken for both heating and cooling in both subsonic and supersonic flow. [20%]

(b) In a certain type of power generating plant, the exhaust from a gas turbine passes through a long pipe of constant diameter 0.3 m that forms part of a heat exchanger. At the inlet to the pipe, the Mach number is 0.4 and the stagnation temperature is 750 K. At the exit from the pipe, the stagnation temperature is 340 K and the static pressure is 1 bar. The pipe is frictionless, and the exhaust gas has the same properties as air. Calculate:

(i) the Mach number at the exit of the pipe; [20%]

(ii) the mass flow rate of exhaust gas; [30%]

(iii) the total amount of heat extracted from the exhaust gas; [10%]

(iv) the velocities at the inlet and the exit of the pipe. [20%]

3 A shock tube consists of a straight pipe of uniform cross-section, divided into two sections by a thin diaphragm. Initially the tube to the left of the diaphragm contains air at a pressure of 3 bar, while the remainder of the tube contains air at 1 bar. The temperature throughout is 300 K. Both ends of the tube are closed. At a certain time the diaphragm bursts instantaneously.

(a) Draw a space-time diagram for the flow development in the right-hand part of the tube, showing the wave reflection from the closed end and at least one particle path. [20%]

(b) Calculate the strength of the initial shock wave. [10%]

(c) Calculate the velocity of the air and the speed of sound behind the initial shock wave. [40%]

(d) Calculate the strength of the reflected shock wave and the temperature of the air behind it. [30%]

4 A bump is to be tested in a supersonic wind tunnel as sketched in Fig. 1. The area ratio A_w/A_s = 1.0738. You may assume that the roof of the expansion section is correctly contoured to prevent reflections entering the working section.

(a) If the supply Mach number, M_s , is 1.082, calculate the Mach number of the flow in the working section upstream of the bump. [20%]

(b) Carefully draw and label the flow features you expect to see through the tunnel from the supply to the crest of the bump, assuming that the flow remains supersonic. [20%]

(c) Using the field method with a 1° discretisation, calculate the Mach number and flow direction in each cell in the expansion section. [30%]

(d) Carefully draw a labelled sketch of the flow in the vicinity of the front half of the bump for each of the cases:

(i) where the ramp is at an angle of 6° to the floor;

(ii) where the ramp is at an angle of 8° to the floor. [30%]

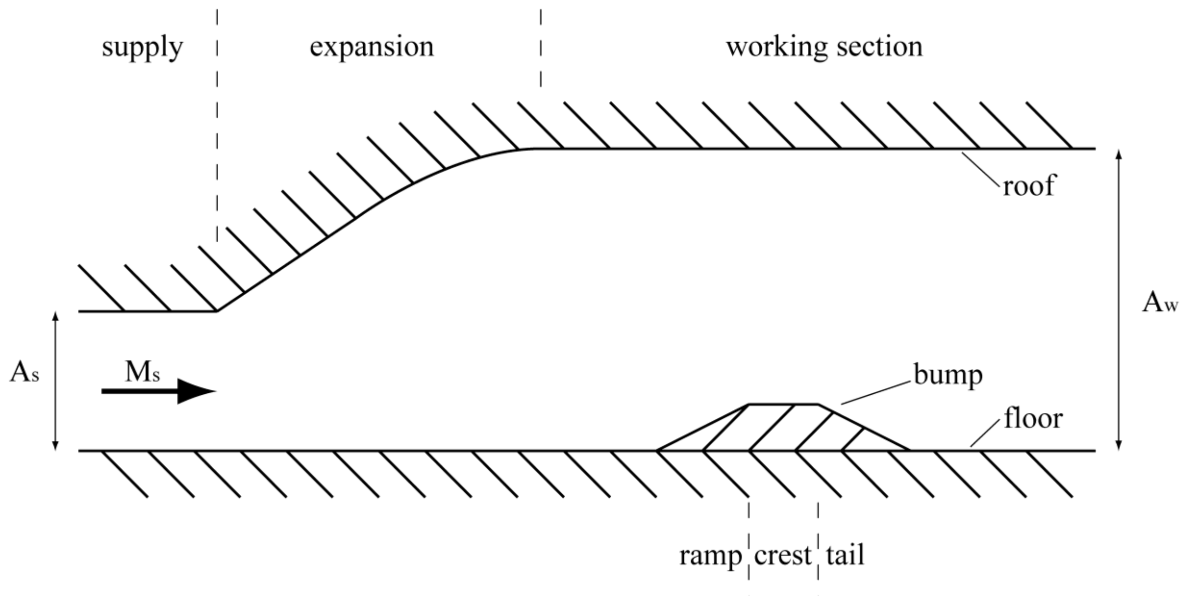

Fig. 1 (not to scale)

5 (a) By considering the flow parallel and normal to an oblique shock wave, show that the density ratio across the shock may be written as:

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) M_1^2 \sin^2 \beta}{2\left[1+\frac{\gamma-1}{2} M_1^2 \sin^2 \beta\right]}$$

[15%]

(b) If a flow with a Mach number of 1.50 is deflected by 8º, what possible values of the shock angle, β , result? Briefly explain:

(i) why there are two analytically valid values;

(ii) how you would predict which solution will occur in practice. [20%]

(c) On a piece of graph paper, plot a Pressure Recovery diagram (i.e. the ratio of stagnation pressure across the shock system against Mach number) for a simple "pitot" (i.e. single shock) engine intake for an aircraft over the Mach number range $1.00 < M < 1.80$. [25%]

(d) In flight testing, the performance of the intake in part (c) is found to be inadequate. As a remedy, an 8º wedge is installed immediately ahead of the intake, such that any shocks that emanate from it do not enter the intake. Add the pressure recovery plot of this revised intake, over the same Mach number range, to your diagram for part (c). [25%]

(e) By how much does the revised intake alter the *static* pressure ratio of the shock system for $M = 1.8$? [15%]

6 A one-dimensional disturbance u convecting in the $+x$ direction at speed A is described by the wave equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$$

(a) Explain what is meant by artificial dissipation, artificial dispersion and false convection in the solution of difference schemes used to solve this equation. [25%]

(b) In order to solve the equation numerically the following difference scheme is used

$$u_i^{n+1} = u_i^n - \frac{c}{2}\left(u_{i+1}^n - u_{i-1}^n\right) + \frac{c^2}{2}\left(u_{i+1}^n - 2u_i^n + u_{i-1}^n\right)$$

where time $t = n\Delta t$, distance $x = i\Delta x$ and $c = A\,\Delta t/\Delta x$.

Show that this scheme is free of artificial dissipation. [50%]

(c) By considering a sawtooth perturbation of small amplitude ε (i.e. the perturbation alternates between $+\varepsilon$ and $-\varepsilon$ at alternate spatial grid points), determine the range of c for which the scheme is stable. [25%]

7 (a) Find an approximation to the first derivative, du/dx, of the function $u(x) = x^2$ at $x = 3$ by using a discretised domain with a uniform spacing of $\Delta x = 0.1$ and

(i) a first-order backward difference approximation, [15%]

(ii) a central difference approximation. Show that this approximation is second-order accurate. [25%]

(iii) Would it be worthwhile to make the difference scheme third-order accurate? [10%]

(b) For a radial compressor the stage loading coefficient is defined as $\psi = \Delta h_0 / U_{tip}^2$ where U_{tip} is the speed of the blade measured at the tip of the impeller.

(i) Show that the stagnation pressure ratio, p_{02}/p_{01}, across the impeller is given by

$$\frac{p_{02}}{p_{01}} = \left(1 + \eta_{tt}\psi \frac{U_{tip}^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

where the isentropic total-to-total efficiency is η_{tt} and T_{01} is the stagnation temperature at inlet. [20%]

(ii) Comment on the significance of the terms ψ and $U_{tip}^2 / (c_p T_{01})$ in the equation in part b (i). [10%]

(iii) The blade tip speed is 300 ms^{-1}, the inlet stagnation temperature is 300 K, the isentropic total-to-total efficiency is 85% and the impeller blade tips are radial. Assuming that the flow leaves the impeller tip at the blade angle, calculate the stagnation pressure ratio. [20%]

You may assume that the working fluid is a perfect gas with:

R = 287 J kg^{-1}K^{-1}, γ=1.4 and c_p = 1005 J kg^{-1}K^{-1}

8 (a) By using a suitable control volume, or otherwise, show that the components of force associated with flow through a row of stationary turbine blades can be written

$$F_x = (p_2 - p_1)A_x + \dot{m}(v_{x2} - v_{x1}) \quad \text{and} \quad F_\theta = \dot{m}(v_{\theta 2} - v_{\theta 1})$$

where F, p, A, $\dot{m}$, v are the force, pressure, area, mass flow and velocity respectively and the subscripts refer to the axial (x) and tangential (θ) direction. Your answer should include:

(i) definitions of A_x and $\dot{m}$;

(ii) whether F is the force on the flow or on the blade;

(iii) all assumptions;

(iv) whether any modifications are needed for a moving blade row. [30%]

(b) An agricultural engineer proposes to use a single stage axial flow turbine described in Table 1 to produce shaft-power from a flow of water supplied by a reservoir with a head of 2 m. Ignoring all pressure losses within the piping and the turbine, for operation at design, determine:

(i) the stagnation enthalpy drop Δh_0 ;

(ii) the blade speed U ;

(iii) the axial velocity of the flow v_x ;

(iv) the mass flow rate $\dot{m}$;

(v) the shaft power produced. [30%]

(c) By using the equations derived in part (a), determine the total force on each rotor blade. [40%]

Geometry	number of rotor blades	10
	tip radius (throughout turbine)	0.10 m
	hub radius (throughout turbine)	0.06 m
At design	stage loading $\left(\Delta h_0 / U^2\right)$	1.2
	Flow coefficient $\left(v_x / U\right)$	0.5
	Exit swirl angle	0°

Table 1

END OF PAPER

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