

Friday 24 April 2015 9.30 to 11.00

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**Module 3A5**

**THERMODYNAMICS AND POWER GENERATION**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**

Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**

CUED approved calculator allowed

Engineering Data Book

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 (a) An adiabatic, steady-flow combustion chamber is supplied with methane (CH<sub>4</sub>) and excess air, both at temperature  $T_1$ . Combustion is complete and the products leave the chamber at temperature  $T_2$ . It may be assumed that 1 mole of air consists of 0.21 moles of oxygen and 0.79 moles of nitrogen, and that all species behave as perfect gases with the same isobaric molar heat capacity  $\bar{c}_p$ . Prove that,

$$T_2 - T_1 = \frac{-\Delta\bar{H}_{T_1}^0}{(1+F)\bar{c}_p}$$

where  $F$  is the molar ratio of air (not O<sub>2</sub>) to methane, and  $\Delta\bar{H}_{T_1}^0$  is the standard molar enthalpy change of reaction at temperature  $T_1$  for the methane combustion reaction,



(b) Derive an expression for the entropy created within the combustion chamber per mole of methane supplied in terms of  $\bar{c}_p$ ,  $T_1$ ,  $T_2$ ,  $F$  and  $\Delta\bar{S}_{T_1}^0$  the standard molar entropy change of reaction at temperature  $T_1$ . It may be assumed that the combustor operates at standard pressure ( $p_0 = 1$  bar) and that the chemical species CH<sub>4</sub>, O<sub>2</sub>, N<sub>2</sub>, CO<sub>2</sub> and H<sub>2</sub>O are each supplied or withdrawn separately at standard pressure. [30%]

(c) For a gas-phase reaction where all species behave as ideal gases, van 't Hoff's equation can be written,

$$\frac{d \ln(K_p)}{dT} = \frac{\Delta\bar{H}_T^0}{RT^2}$$

where  $K_p$  is the equilibrium constant. If, for a particular reaction,  $\Delta\bar{H}_T^0$  is independent of temperature, prove that the standard molar entropy change of reaction  $\Delta\bar{S}_T^0$  is also independent of temperature. [20%]

(d) In a particular application the combustor forms part of a power plant. The inlet and outlet combustor temperatures are  $T_1 = 800$  K and  $T_2 = 1750$  K, and a suitable average value for  $\bar{c}_p$  is  $30.0 \text{ kJ kmol}^{-1}\text{K}^{-1}$ . The reactants are supplied to the power plant at  $T_0 = 25$  °C and, for the methane combustion reaction,  $\Delta\bar{G}_{T_0}^0 = -800.8 \text{ MJ kmol}^{-1}$  and  $\Delta\bar{H}_{T_0}^0 = -802.3 \text{ MJ kmol}^{-1}$ . Using the assumptions, approximations and results from parts (a), (b) and (c) above, and taking the dead state temperature as  $25$  °C, calculate the maximum rational efficiency of the plant assuming the only irreversibility is that associated with the combustor. [25%]

2 The Dieterici equation of state of an imperfect gas is,

$$p = \frac{RT}{(v-b)} \exp\left(-\frac{a}{RTv}\right)$$

where  $p$  is the pressure,  $T$  is the absolute temperature,  $v$  is the specific volume,  $R$  is the specific gas constant, and  $a$  and  $b$  are constants.

(a) Write down two mathematical conditions at the critical point that must be satisfied by the isotherm passing through that point. [10%]

(b) Apply these conditions to show that, for Dieterici's equation, the critical pressure, temperature and specific volume,  $p_c$ ,  $T_c$  and  $v_c$ , are related by,

$$\frac{p_c v_c}{RT_c} = 2e^{-2} = 0.271 \quad [35\%]$$

(c) Using a scaling based on the critical properties, derive a universal or 'corresponding states' form for Dieterici's equation of state. [15%]

(d) State the three conditions that must be satisfied by the liquid and vapour phases of a pure substance coexisting at thermodynamic equilibrium. Show that one of these conditions is equivalent to the statement,

$$\int_f^g v dp = 0$$

where  $f$  and  $g$  stand for the wet saturated liquid and dry saturated vapour states, respectively. Hence, show that,

$$\int_f^g p dv - p_{sat}(T)(v_g - v_f) = 0$$

where  $p_{sat}(T)$  is the saturated vapour pressure at temperature  $T$ . [25%]

(e) Interpret the second equation of part (d) graphically with reference to a sketch of the  $p$ - $v$  diagram for Dieterici's equation of state. [15%]

3 Where necessary, use the properties of steam given in Table 1 on the next page.

(a) Explain how feedheating can be used to raise the efficiency of a steam cycle. [10%]

(b) A temperature-entropy diagram for a steam cycle, with reheat, is shown in Fig. 1(a). The condenser pressure is 0.04 bar and the flow leaves the condenser as saturated liquid. The boiler pressure is 150 bar and the reheat pressure is 40 bar. Steam leaving both the boiler and the reheater is superheated to 550 °C. Both turbines may be modelled as isentropic and the feed pump work neglected. Stating any other assumptions, evaluate the maximum possible cycle efficiency for the following configurations:

(i) without feedheating,

(ii) with feedheating using steam bled from the turbine after the reheater. [35%]

(c) Two direct-contact feedheaters are fitted to the cycle, as indicated in Fig. 1(b). The feedheaters extract fractions  $x$  and  $y$  (of the mass flowrate of steam leaving the reheater) from the turbine at pressures of 20 bar and 5 bar respectively. In each case, the flow leaving the feedheater is saturated liquid. Stating any assumptions made, evaluate the efficiency of the cycle. [35%]

(d) Describe an undesirable consequence of feedheating on the temperature of the exhaust flue gas leaving the boiler. What feature can be added to the plant to mitigate this? [20%]

(cont.)

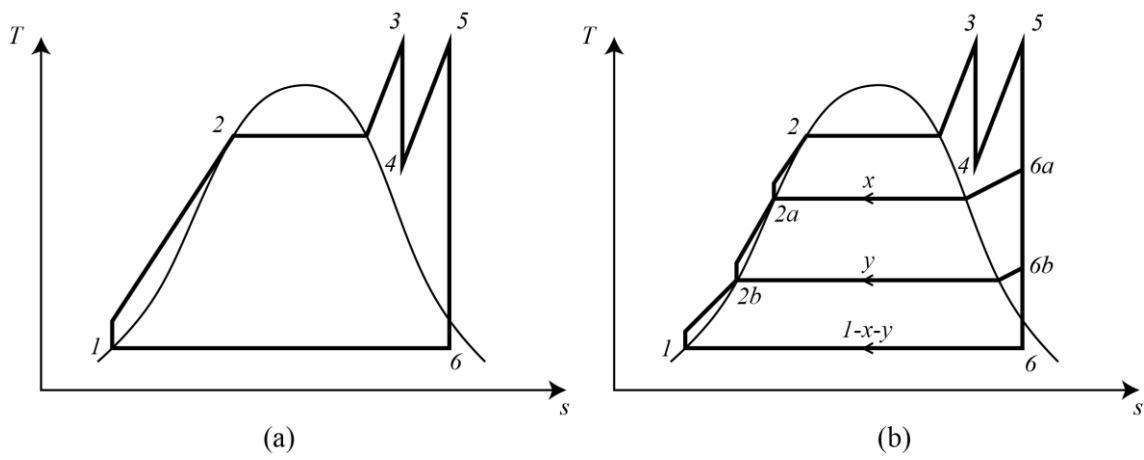


Figure 1

State 4	$h = 3060 \text{ kJ kg}^{-1}$
State 6	$h = 2180 \text{ kJ kg}^{-1}$
State 6a	$h = 3330 \text{ kJ kg}^{-1}$
State 6b	$h = 2950 \text{ kJ kg}^{-1}$

Table 1

4 (a) Explain why polytropic efficiency is often preferred to isentropic efficiency for the performance analysis of compressors and turbines. [15%]

(b) A semi-perfect gas is expanded, with polytropic efficiency  $\eta$ , from state 1 to state 2, in a turbine. Show that,

$$\eta R \ln \left( \frac{p_2}{p_1} \right) = \int_{T_1}^{T_2} \frac{c_p}{T} dT$$

where  $R$  is the specific gas constant and  $c_p(T)$  is the specific heat capacity at constant pressure. [20%]

(c) Air at 45 bar and 1800 K enters a turbine where it is expanded with a polytropic efficiency of 0.90. The flow leaves the turbine at a pressure of 1 bar. The air may be assumed to be semi-perfect with  $R = 0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$  and,

$$c_p = c_{p0} \left( \frac{T}{T_0} \right)^n$$

where  $c_{p0} = 1.010 \text{ kJ kg}^{-1} \text{ K}^{-1}$ ,  $T_0 = 298.15 \text{ K}$  and  $n = 0.12$ . Evaluate:

(i) the specific work output from the turbine; [30%]

(ii) the isentropic efficiency of the turbine. [15%]

(d) Compare the role of aero-derivative gas turbines to gas turbines typically used in combined cycle plants. Describe two modifications to the aero-derivative cycle that can be used to raise cycle efficiency. [20%]

**END OF PAPER**

## ANSWERS

1. (b)  $\Delta \bar{s}_{irrev} = \Delta \bar{s}_{T_1}^0 + (1+F)\bar{c}_p \ln\left(\frac{T_2}{T_1}\right)$
- (d)  $\eta_{rat,max} = 0.756$
2. (a)  $\left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0$
- (c)  $p_R = \frac{e^2 T_R}{(2v_R - 1)} \exp\left(-\frac{2}{T_R v_R}\right)$  where  $p_R = p/p_C$ ,  $T_R = T/T_C$ ,  $v_R = v/v_C$
- (e) The equal-area 'Maxwell construction' ensuring that  $g_f = g_g$ .
3. (b) (i) 0.460  
(ii) 0.542
- (c) 0.503
4. (c) (i) 1214 kJ kg<sup>-1</sup>  
(ii) 0.937

J.B. Young & G. Pullan

ENGINEERING TRIPOS PART IIA 2015

MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION

ANSWERS

1. (b)  $\Delta \bar{s}_{irrev} = \Delta \bar{s}_{T_1}^0 + (1+F)\bar{c}_p \ln\left(\frac{T_2}{T_1}\right)$
- (d)  $\eta_{rat,max} = 0.756$
2. (a)  $\left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0$
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- (e) The equal-area ‘Maxwell construction’ ensuring that  $g_f = g_g$ .
3. (b) (i) 0.460  
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