EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 3 May 2017   2 to 3.30

Module 3A5

THERMODYNAMICS AND POWER GENERATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Derive the Maxwell relation

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T$$

where the symbols have their usual meanings. [10%]

(b) The characteristic equation of state in the Helmholtz formulation for a particular *non-ideal* gas is given by

$$f(T,v) = \alpha\left(T - T_r - T\ln\frac{T}{T_r}\right) - \frac{\beta}{2}(T-T_r)^2 - RT\ln\left(\frac{v-b}{v_r-b}\right)$$

where α, β, b, R, T_r and v_r are constants, and other symbols have their usual meanings. Find the relationship between pressure, temperature and specific volume for the gas. Determine also the dependence of specific internal energy on T and v and hence derive an expression for the specific heat capacity at constant volume. Describe briefly the molecular model of the gas and the physical significance of each of the quantities b, T_r and v_r. [45%]

(c) The gas of part (b) flows steadily through a partially closed valve at a rate of 0.75 kg s^{-1}. The upstream pressure and temperature are 50 bar and 450 K respectively, and the pressure drop through the valve is 10 bar. Stating your assumptions and using the data given below, determine the exergetic loss rate as a result of flow through the valve. What would be the exergetic loss rate for an ideal gas with the same R undergoing a similar process with the same upstream conditions and same pressure drop? Assume that the environment temperature is $T_0 = 298.15$ K. [45%]

Data for part (c):

$R = 0.4$ kJ kg^{-1} K^{-1} $\alpha = 1.0$ kJ kg^{-1} K^{-1} $\beta = 0.005$ kJ kg^{-1} K^{-2}

$b = 0.05$ m^3 kg^{-1} $T_r = 300$ K $v_r = 1.05$ m^3 kg^{-1}

2 (a) The van 't Hoff equation for the variation of the equilibrium coefficient with temperature may be written

$$\frac{d\left(\ln K_p\right)}{d(1/T)} = -\frac{\Delta\widetilde{H}_0^T}{\widetilde{R}}$$

where $\Delta\widetilde{H}_0^T$ is the standard molar enthalpy of reaction and $\widetilde{R}$ is the molar gas constant. Using this equation or otherwise, explain how an increase in temperature affects the equilibrium of the reaction

$$\tfrac{1}{2}N_2 + \tfrac{3}{2}H_2 \rightleftharpoons NH_3$$ [10%]

Note that data for this reaction is available in the data book.

(b) 0.5 kmols of N_2 and 1.5 kmols of H_2 are contained in an insulated rigid vessel at an initial pressure and temperature of $P_0 = 1$ bar and $T_0 = 298.15$ K. A catalyst is introduced and the reaction described in part (a) occurs such that x kmols of NH_3 are formed once equilibrium is reached. Show that the final temperature T is related to x by

$$\frac{x}{(1-x)^2} = CK_p(T)\frac{T}{T_0}$$

and find the value of the coefficient C. Treat all components as ideal gases. [35%]

(c) Determine a second relationship between x and T and hence show that the final temperature is approximately 500 K. You may assume that the average isobaric molar heat capacity of the reaction products is $\widetilde{C}_p = 33$ kJ kmol^{-1} K^{-1}. [40%]

(d) Keeping the temperature constant, the pressure within the vessel is now doubled by the addition of an appropriate quantity of argon gas. Determine the new value of x . [15%]

 (TURN OVER

3 (a) Explain, with reference to the thermodynamic cycle, why blade cooling can be beneficial to the performance of a gas turbine. [15%]

(b) A land-based gas turbine has an overall pressure ratio r_p and operates with air as the working fluid. The *polytropic* efficiencies of the compressor and turbine are the same and are equal to η. For cooling purposes, a fraction m of the air entering the compressor is removed from the compressor at pressure p_{m}, where $p_{\text{m}}/p_1 = \alpha r_p$ and p_1 is the compressor inlet pressure. This cooling air is added to the flow in the turbine at the same pressure, p_{m}. The ratio of turbine inlet temperature to compressor inlet temperature is θ. It may be assumed that there is no pressure loss in the combustor and that the turbine exhausts to atmospheric pressure. The air may be modelled as a perfect gas with ratio of specific heats $\gamma = 1.4$ and specific heat capacity at constant pressure $c_p = 1.01$ kJ kg^{-1}K^{-1}. The mass flow rate of fuel may be neglected. The dead state temperature is the same as the compressor inlet temperature, $T_1 = 298.15$ K.

For a configuration in which $r_p = 20$, $\eta = 0.9$, $m = 0.1$, $\alpha = 0.8$ and $\theta = 5$:

(i) calculate, stating any assumptions, the temperature of the air after the mixing of the coolant and main turbine flow has taken place (before the mixed out air enters the rest of the turbine); [20%]

(ii) evaluate the lost work due to this mixing process (expressed as a fraction of the turbine work output without cooling), and indicate this loss on a temperature-entropy diagram of the cycle; [40%]

(iii) obtain the total work output from the turbine (expressed as a fraction of the turbine work output without cooling). [15%]

(c) In addition to the loss of part (b), describe two other sources of irreversibility that are likely to be associated with gas turbine cooling. [10%]

4 A steam cycle for a nuclear Pressurised Water Reactor (PWR) has the following specification. The condenser pressure is 0.04 bar. The feed pump raises the water pressure to 60 bar. The steam enters the High Pressure (HP) turbine dry-saturated and leaves the HP turbine at a pressure of 10 bar. The steam then enters a reheater where its temperature is raised to 250°C (at 10 bar) before entering the low pressure (LP) turbine, which exhausts to the condenser pressure.

(a) Sketch the steam cycle on a temperature-entropy diagram. Discuss the limitations on peak steam temperature and the benefits of the reheater. [25%]

(b) The isentropic efficiency of the HP turbine is 85%. Evaluate the specific work output of the HP turbine. [25%]

(c) The reheater is a heat exchanger that may be assumed to have no pressure losses. The hot side of the heat exchanger is fed by steam extracted from the HP inlet flow; this flow enters as dry-saturated steam and leaves as saturated liquid before being returned to the boiler at the same pressure. Evaluate the fraction m of the mass flow that is extracted at HP turbine inlet. [10%]

(d) Steam is extracted at several points in the LP turbine and a train of feedheaters is used to raise the temperature of the feedwater to 200°C, wet-saturated. Stating any assumptions, calculate the maximum work that could be extracted from the LP turbine. Evaluate the cycle efficiency under these conditions. [25%]

(e) With reference to a sketch on a temperature-entropy diagram, compare the steam cycle in this PWR design to one that would be typical for an Advanced Gas-cooled Reactor (AGR). [15%]

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Numerical answers:

1 (c) 44.43kW, 19.96 kW

2 (b) $C = \sqrt{27}/8$; (d) x remains at 0.206 kmol

3 (b)(i) 1339K; (b)(ii) 0.63%; (b)(iii) 0.95

4 (b) 274 kJ/kg; (c) 0.216; (d) 702.5 kJ/kg, 0.413