EGT2
ENGINEERING TRIPOS PART IIA

Friday 1 May 2015   9.30 to 11

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A flammable gas sensor consists of a porous diffusion layer of thickness δ over a heated catalytic plate (Fig. 1). A gas stream with a molar bulk concentration c_b of the flammable species flows over the surface of the sensor, where the concentration is c_s. The species diffuses through the layer onto a heated plate, where it reacts exothermically, leading to a temperature rise, which is measured. The mass convection coefficient over the sensor is h_m, and the diffusion coefficient of the species through the porous layer is D. The rate of reaction at the plate is $\Omega = Ac_c$, where A is a constant, and c_c is the molar concentration at the catalyst surface.

(a) Consider the device in steady state operation.

(i) Obtain an expression for the concentrations c_c and c_s, as a function of the bulk gas concentration and the parameters given. [30%]

(ii) Show that the molar flux through the sensor reduces to $j = Ac_b$ at the limit of low reaction rate, and to $j = c_b \dfrac{1}{1/h_m + \delta/D}$ at the limit of high reaction rate. [25%]

(b) Consider the device operating under a sudden change in concentration, and at the limit of high reaction rate, as described in part (a)(ii). The boundary conditions are $c(\delta,t) = c_{s1}$ and $c(0,t) = 0$ for $t > 0$, and the initial concentrations are zero throughout the domain.

(i) Estimate the response time of the sensor, as the time for the reactant to get to the wall at $x = 0$, as a function of the parameters given. [15%]

(ii) Show by substitution that the solution for the concentration profile through the layer and the boundary conditions are satisfied by equations of the form: [20%]

$$c = C_n \exp(-k_n^2 D/\delta^2 t)\sin(k_n x/\delta)$$

(iii) Explain what are allowable values for k_n. [10%]

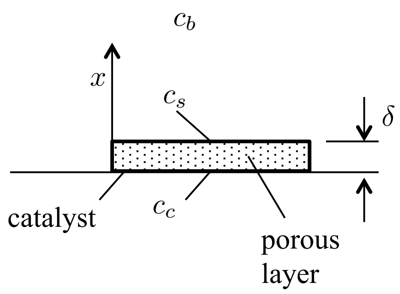

Fig. 1

2 A finned heat exchanger is formed by stacking flow channels of height H, width W, length L, and wall thickness δ, as shown in Fig. 2(a). The inlet temperature of the hot air is $T_{h,i}$, and the outlet temperature is $T_{h,o}$. The air mass flow rate is $\dot{m}$, and the specific heat at constant pressure, c_p. The conductivity of the solid surfaces is λ. The operation is in steady state, and the flow can be considered fully developed throughout, with a constant heat transfer coefficient, h, for all surfaces. The number of channels is large, so that the analysis can be done for a unit element consisting of a half channel about the symmetry axis of the channel walls, as shown Fig. 2(b). Heat transfer to the end walls can be neglected. The temperature of the surface at the mid-point between stacks is T_s, and the temperature gradients at all symmetry planes are zero.

(a) Consider the analysis of the unit element defined in Fig. 2(b). Ignoring the fins, show that the total heat transfer from the air to a surface of area WL to the symmetry plane at a fixed temperature T_s within the heat exchanger is given by $q = UWL\theta$, where $\theta = \dfrac{(T_{h,o} - T_{h,i})}{\ln[(T_{h,o} - T_s)/(T_{h,i} - T_s)]}$, and U is the overall heat transfer coefficient. Determine U as a function of the parameters given. [30%]

(b) Considering the additional transfer by the vertical fins, determine the total heat transfer to the unit element in terms of the geometric parameters (W, δ, H and L), θ and flow properties. Assume that the fin effectiveness, defined as the ratio of the actual heat transfer to that of a surface at a uniform base temperature, is given as $\eta = [\tanh(m\,L_f)]/(m\,L_f)$ for an adiabatic tip, where L_f is the fin length, $m = \sqrt{hP_f/(\lambda A_c)}$, A_c is the cross-sectional area of the fin, and P_f its perimeter. [30%]

(c) Determine the overall heat transfer for a system consisting of N rectangular channels, neglecting end effects. [10%]

(d) Consider now an assembly with rows of stacks of identical air units in cross flow, organised so that the surfaces line up at the stack symmetry planes, with a heat exchange area W^2. Show that the temperature rise ΔT of the cold and hot sides across the unit are equal and opposite, and obtain an expression for ΔT as a function of the geometric and operational parameters given. [30%]

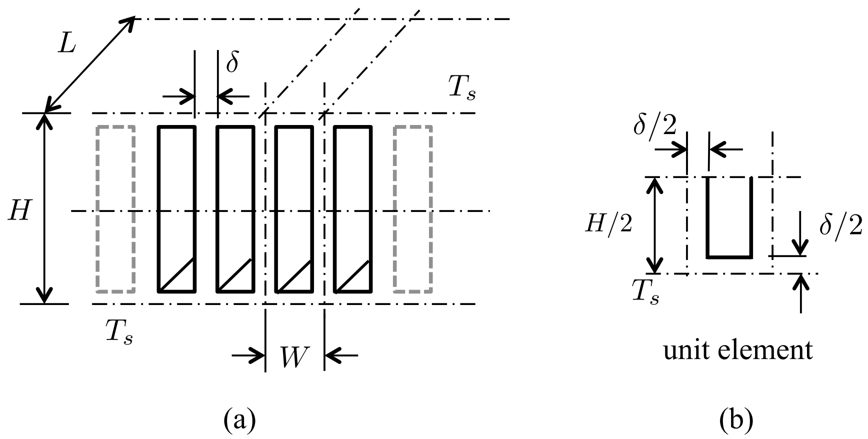

Fig. 2

3 A flow velocity sensor works by placing a heated film aligned with a one-dimensional flow with free stream velocity U, and measuring the necessary rate of heat transfer to the film required to maintain it at a given temperature, as shown in Fig. 3. The film surface, of length 0.005 m along the flow, is maintained at a higher temperature than the free stream, via a feedback circuit. The local Nusselt number over the surface of the film is assumed to be given by one of the following correlations, which are valid for flow over flat plates:

$$Nu_x = 0.332\, Pr^{1/3} Re_x^{1/2} \qquad \text{laminar flow, } Re_x < 10{,}000$$

$$Nu_x = 0.0385\, Pr^{1/3} Re_x^{4/5} \qquad \text{turbulent flow, } Re_x \geq 10{,}000$$

(a) Obtain an expression for the mean Nusselt number over a length L , $\overline{Nu}_L = \dfrac{\overline{h}L}{\lambda}$, for the two flow regimes, as a function of Pr and Re_L. [15%]

(b) Determine the local Re_L, Nu_L and mean $\overline{Nu}_L$ numbers for the sensor length L, for the cases on the table below. Be mindful of the turbulent transition. [35%]

Property	Case			
	A	B	C	D
U (m/s)	0.1	10	0.1	10
Fluid	air		water	
ν (m^2/s)	1.47×10^{-5}		1.15×10^{-6}	
Pr (-)	0.7		8.12	

(c) Sketch the evolution of the thermal and velocity boundary layer thicknesses in the region $0 < x < L$ for cases A-D, justifying the relative magnitudes and highlighting notable features. [20%]

(d) Estimate the uncertainty in velocity measurement $\Delta U/U$ for a given mean heat flux measurement uncertainty, $\Delta q/q$. Discuss what determines the relative measurement accuracy. [15%]

(e) Are the thin boundary layer approximations valid under conditions A-D? What alternatives could be used to relate the heat flux and flow velocity for different sensor geometries? [15%]

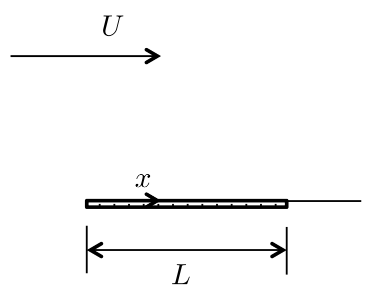

Fig. 3

 (TURN OVER

4 A solid slab of material with thermal conductivity λ has the quarter cylindrical cross section of radius R indicated in Fig. 4. The material generates heat at a rate of $\dot{g}$ per unit volume.

(a) Provide a sketch and derive the steady state energy conservation equation in terms of the temperature T of the material, for a cylindrical differential element $dr \times r\, d\theta \times dz$. [30%]

(b) Assuming that the slab is semi-infinite in the z direction, with negligible temperature gradients in z and θ, that the slab is perfectly insulated at $\theta = 0$ and $\theta = \pi/2$, and cooled at the outer boundary, subject to a convection coefficient h and a surrounding fluid temperature T_∞ , show that the solution to the temperature field is given by: [30%]

$$T = T_\infty + \frac{\dot{g}R}{2h} + \frac{\dot{g}R^2}{4\lambda}(1 - \frac{r^2}{R^2})$$

(c) Now assume that the boundary conditions at all three walls are convective, with the same convection coefficient h and a surrounding fluid temperature T_∞ . Provide a sketch and discretise the governing equation in cylindrical coordinates for internal points, using second order central differencing. [20%]

(d) Provide the discretised equations for external points along the three external walls. Discuss how the system can be organised in matrices to obtain a solution in a grid with N radial and M angular grid points. [20%]

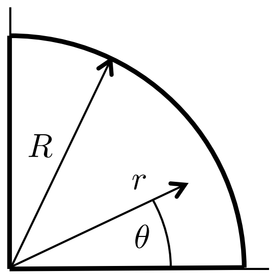

Fig. 4

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Answers

1.

(a)(i) $c_b - c_c = Ac_c(\frac{1}{h_m} + \frac{\delta}{D})$

$c_s = c_b \frac{1/A + \delta/D}{1/A + 1/h_m + \delta/D}$

(a)(ii) -

(b)(i) $\tau = \delta^2/D$

(b)(ii) -

(b)(iii) $k_n = \pi(n + 1/2)$

2.

(a) $1/U = 1/h + (\delta/2)/\lambda$

(b) $q_u = UWL\theta_l[1 + \frac{\delta}{L}(\eta - 1)]$, $L_f = H/2$, $m = \sqrt{2h/\lambda\delta}$

(c) $Q = 2Nq_u$

(d) $\Delta T_h = -\Delta T_c$

$\Delta(T_h - T_c) = -\frac{U}{2}W^2[1 + \frac{\delta}{L}(\eta - 1)](T_h - T_c)$

4.

(a) For the laminar case,

$\overline{Nu}_{L,l} = (2)0.332 Pr^{1/3} Re_L^{1/2}$

For the turbulent case,

$\overline{Nu}_{L,t} = \frac{5}{4} 0.0385 Pr^{1/3} Re_L^{4/5}$

Considering the transition at ℓ

$\overline{Nu}_L = \frac{\overline{h}_L L}{\lambda} = \frac{Nu_{\ell,l}}{m} + \frac{Nu_{L,t}}{m'} - \frac{Nu_{\ell,t}}{m'}$

(b)

	Air		Water	
Case	A	B	C	D
U (m/s)	1	10	1	10
Re_L (-)	34	3401	435	43478
Nu_L	1.7	17.2	13.9	397 (t)
$Nu_{\ell,t}$				123
$Nu_{\ell,l}$				66.7
$\overline{Nu}_L$	3.4	34	28	477

(c) $\dfrac{\Delta U}{U} = \frac{1}{m}\dfrac{\Delta q}{q}$, where $m = 1/2$ for laminar and $m = 4/5$ for turbulent.

(d) -

5.

(a) -

(b) -

(c) -

(d) -