Version SH/6

EGT2
ENGINEERING TRIPOS PART IIA

Friday 28 April 2017 9.30 to 11

Module 3A6

HEAT AND MASS TRANSFER

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Two rooms containing hot and cold gases at temperatures $T_h > T_c$ are separated by a wall of thickness L, height H and width W (Fig. 1). The pressure in each room is hydrostatic, with gravity acceleration g as noted. The mean pressure is the same on either side of the wall. Gaps of height $D \ll H$ on top and bottom allow flow in either direction. Assume that the bulk velocity across the gaps is given by $U_b = \dfrac{D^2}{12}\dfrac{\Delta p}{\rho \nu L}$, where ρ is the mean density within the channel, ν the kinematic viscosity, and Δp the pressure difference across the channel.

(a) Determine the pressure difference across the top and bottom gaps, and show that the mass flow rate $\dot{m}$ leaking across each channel is equal to: $\dot{m} = (\rho_c - \rho_h)\dfrac{gD^3WH}{24\nu L}$. [20%]

(b) Starting from the governing equations for the vertical thermal boundary layer below, featuring a volumetric expansion coefficient β, and thermal diffusivity α, apply a scaling analysis to estimate the mass flow rate induced by natural convection, as follows:

$$\text{Mass:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\text{Vertical momentum:} \quad u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \nu\frac{\partial^2 v}{\partial x^2} + g\beta(T - T_\infty)$$

$$\text{Energy:} \quad u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial x^2}$$

(i) Sketch the velocity and temperature profiles along the wall for the cold and hot sides: which way does the air flow? [10%]

(ii) Using mass and energy conservation equations, estimate the vertical velocity v induced by natural convection as a function of the thermal boundary layer thickness δ_T, the height H and the thermal diffusivity, α. [20%]

(iii) Assuming that the leading terms in the vertical momentum equation are friction and buoyancy, show that: [25%]

$$\frac{\delta_T}{H} \sim \left(\frac{\alpha\nu}{gH^3\beta(T_h - T_c)}\right)^{1/4}$$

(c) Using the estimates of v and δ_T , determine the ratio of the estimated mass flow rates induced by natural convection compared to those generated by leakage in part (a). [25%]

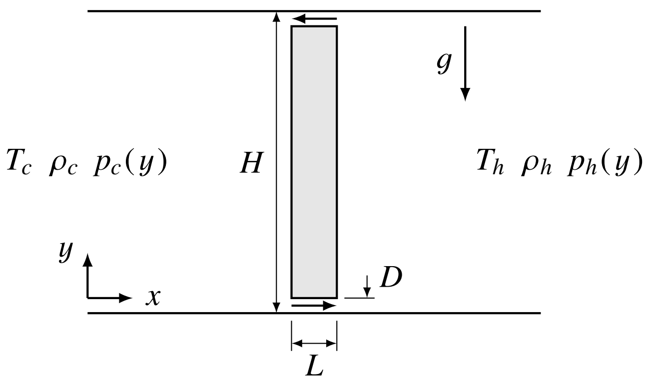

Fig. 1

2 A two-dimensional mass exchange channel of length L (Fig. 2) selectively extracts a species from a dilute mixture flowing through channel A into the lower concentration stream B, via a porous membrane through which only the species of interest is allowed to cross. The species flow resistance of the membrane is negligible. Both mass fractions Y_A and Y_B are sufficiently low that the overall mass flow rates per unit width $\dot{m}_A$ and $\dot{m}_B$, and the density of both streams, ρ, remain unchanged. The convective mass transfer coefficients per unit area for each side of the channel are h_A and h_B respectively, in units of length per unit time. The boundary conditions for the concentrations are $Y_A(0) = Y_{A0}$, $Y_A(L) = 0.5\, Y_{A0}$ and $Y_B(L) = 0$, and we define the ratio $\beta = \dot{m}_A/\dot{m}_B$.

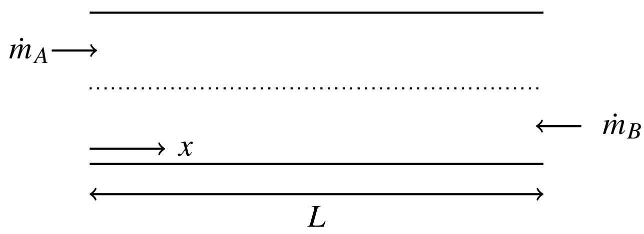

Fig. 2

(a) Starting from a differential element along x, derive an expression for the local species mass flux , j , across the membrane, as a function of $\Delta Y = (Y_A - Y_B)$, density, and mass convection coefficients. [10%]

(b) Show that the total flow rate of the species of interest across the membrane per unit width, $J = \int_0^L j\, dx$ is given as:

$$J = \rho U L \frac{\Delta Y(0) - \Delta Y(L)}{\ln[\Delta Y(0)/\Delta Y(L)]}$$

where $U = \left[\frac{1}{h_A} + \frac{1}{h_B}\right]^{-1}$. [30%]

(c) Obtain an expression for $J/(\rho U L)$ for the boundary conditions given, as a function of β and Y_{A0} . [25%]

(d) Obtain an expression for $j/(\rho U)$ as a function of x/L , $\rho U L/\dot{m}_A$, Y_{A0} and β . [15%]

(e) Sketch j, Y_A and Y_B as a function of x/L, for $0 < \beta < 1$, and identify any relevant features. [20%]

3 (a) Starting from a differential cylindrical element $r\,\mathrm{d}r\,\mathrm{d}\theta$ in a solid material, derive the two-dimensional energy conservation equation, where heat is transferred by conduction and generated at a rate $\dot{g}$ per unit volume :

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T + \frac{\dot{g}}{\rho c}$$

The material has thermal diffusivity α , density ρ and specific heat c . Include a sketch of the differential elements and all steps in your derivation. [30%]

(b) For a cylindrical coordinate system, in which $\dot{g} = 0$, determine what condition or conditions functions $f(t)$ and $\beta(t)$ must obey in order to satisfy energy conservation for the following temperature field: [30%]

$$T(r,t) = f(t)\exp(-\beta(t)r^2)$$

(c) For the temperature field:

$$T(r,t) = \frac{A}{\alpha t}\exp\left(-\frac{r^2}{4\alpha t}\right)$$

(i) Sketch the temperature field contours, and the corresponding heat flux lines for a given time t. [10%]

(ii) Sketch the temperature profile as a function of distance r and time t . [10%]

(iii) Show by integration that the energy in the entire field is conserved, and interpret the constant A. [20%]

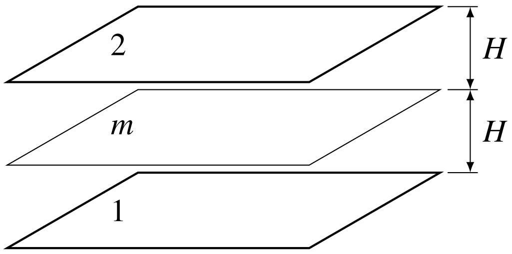

Fig. 3

4 Two identical rectangular parallel plates, represented as surfaces 1 and 2 in Fig. 3, are separated by a semi-transparent window m of the same dimensions, placed at the mid-point between plates 1 and 2. Surfaces 1 and 2 are opaque, with emissivities $\epsilon_1 = 0.8$ and $\epsilon_2 = 0.7$, and all surfaces can be considered diffuse. The view factors for the rectangular plates are $F_{12} = 0.3$ and $F_{1m} = 0.4$, respectively. The system is at steady state, the temperatures of surfaces 1 and 2 are T_1 and T_2, respectively, and the window temperature is T_m. The plates are surrounded by a medium at ambient temperature T_∞. The window transmissivity is τ_m, and its reflectivity, $\rho_m = 0$.

(a) Show that the net radiative flux between surfaces 1 and 2 transmitted through the window is given by $q_{12,\tau} = \dfrac{J_1 - J_2}{1/(F_{12}\tau_m)}$, where J_i is the radiosity of surface i. [15%]

(b) Sketch the network for radiation exchange between all surfaces, including the surroundings, clearly labelling all resistances. [30%]

(c) For the following questions, neglect the exchange with the surroundings.

(i) Obtain an expression for the total radiative flux q_{12} between surfaces 1 and 2, as a function of the blackbody radiation emissive power $E_{b,1}$ and $E_{b,2}$, for $\tau_m = 0.6$. [25%]

(ii) Consider a case where, for $\lambda < \lambda_0$, $\tau_m = 0.5$, and for $\lambda > \lambda_0$, $\tau_m = 0.6$. Write an expression for the total heat flux q_{12} as a function of the new parameters, and explain how you would obtain the final value of q_{12}. [30%]

END OF PAPER