EGT2
ENGINEERING TRIPOS PART IIA

Monday 18 April 2016 2 to 3.30

Module 3B5

SEMICONDUCTOR ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An electron is confined in a one-dimensional potential well given by

$$V(x) = k\frac{x^2}{2}$$

where k is the spring constant and x is position in the one dimension. This potential energy function is commonly called a *harmonic oscillator* potential and is plotted in Fig. 1. The wave function of the quantum mechanical ground (lowest energy) state of the electron is described by

$$\Psi_0(x,t) = A\exp\left(-\frac{\sqrt{km}}{2\hbar}x^2\right)\exp(-j\omega t)$$

where m is the mass of the electron, A and ω are constants, and t is time.

(a) Show that $\omega = \frac{1}{2}\sqrt{k/m}$ in order for this wave function to be a valid solution of the time-dependent Schrödinger equation. [30%]

(b) Given $\omega = \frac{1}{2}\sqrt{k/m}$, derive an expression for the total energy, E_0, of the electron in the ground state. [10%]

(c) The uncertainty in the electron's position is $\Delta x = 2\hbar^{1/2}/(km)^{1/4}$. What is the minimum kinetic energy, T, of the electron if Heisenberg's uncertainty principle is to be satisfied? How does this value compare with the ground state energy, E_0, calculated in part (b)? Comment on your results. [20%]

(d) The full solution of the Schrödinger equation for the harmonic oscillator well shows that the electron can only take discrete values of energy given by $E_n = (2n+1)E_0$ for $n = 0, 1, 2, ...$ Sketch the wave functions corresponding to energy levels E_0, E_1 and E_2 onto a plot of the potential energy function as shown in Fig. 1. [25%]

(e) Two identical harmonic oscillator potential wells, each occupied by one electron, are separated by a distance, d. Explain what happens to the electron energy levels as d is reduced to the nanometre scale. [15%]

Note: The time-dependent Schrödinger equation in one dimension is

$$-\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi = j\hbar\frac{\partial}{\partial t}\Psi$$

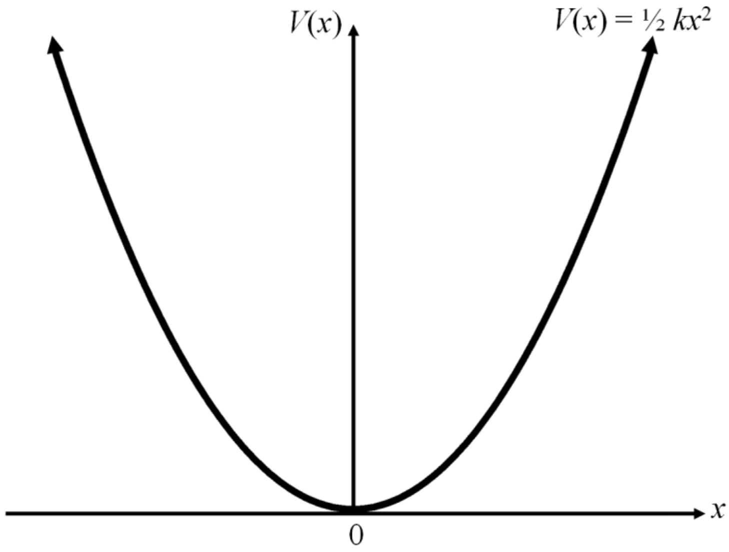

Fig. 1

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2 (a) Why must Fermi–Dirac statistics be used to determine the probability that an electron occupies a particular quantum mechanical state, rather than Boltzmann statistics? [15%]

(b) Explain the meaning of the *Fermi energy*, E_F. [15%]

(c) The density of states in the valence band of a semiconductor is given by

$$g(E)\mathrm{d}E = \frac{V}{2\pi^2\hbar^3}(2m_h^*)^{\frac{3}{2}}(E_V - E)^{\frac{1}{2}}\mathrm{d}E$$

where E is the energy, E_V is the valence band energy, and m_h^* is the hole effective mass. Show that when $E_F - E_V \gg kT$, the hole concentration, p, is given by

$$p = N_V \mathrm{e}^{\left(\frac{-(E_F - E_V)}{kT}\right)}$$

where

$$N_V = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{\frac{3}{2}}.$$

Equations that may be useful are given on page 5. [30%]

(d) A GaAs sample is intentionally p-doped with zinc (Zn) atoms to a density of $N_{Zn} = 1.0 \times 10^{17}\ \mathrm{cm}^{-3}$. GaAs has an effective density of states in the valence band of $N_V = 9.0 \times 10^{18}\ \mathrm{cm}^{-3}$ and an intrinsic carrier density of $n_i = 2.1 \times 10^6\ \mathrm{cm}^{-3}$ at 290 K.

(i) Calculate the position of the Fermi level, E_F, at 290 K. State clearly any assumptions made. [15%]

(ii) Owing to a dirty fabrication environment, silicon (Si) donor impurities contaminate the p-GaAs. The density of Si impurity atoms is $N_{Si} = 2.0 \times 10^{16}\ \mathrm{cm}^{-3}$. The density of Zn acceptors remains at $N_{Zn} = 1.0 \times 10^{17}\ \mathrm{cm}^{-3}$. Explain, with the aid of equations and band diagrams, how the presence of Si donors affects the hole population. Calculate the majority and minority carrier densities in the contaminated p-GaAs. [25%]

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Note:

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

$$\int_0^\infty x^2 \mathrm{e}^{(-\alpha x^2)}\,\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

$$np = n_i^2$$

3 A Schottky barrier diode is fabricated using n-type silicon. The silicon has an electron affinity of 4.05 eV and an effective density of states in the conduction band of 2.8×10^{25} m^{-3}. The metal has a work function of 4.9 eV.

(a) (i) Draw an equilibrium band diagram of the diode. Indicate the built-in potential V_0 in the diagram. [10%]

(ii) Calculate the doping density required to tune V_0 to 0.7 V at room temperature. State all assumptions made. [20%]

(iii) Outline how V_0 could be measured. [10%]

(b) An external forward bias V_a of 0.6 V is applied to the Schottky barrier diode.

(i) Draw a band diagram of the biased junction and indicate the Fermi levels. [10%]

(ii) Calculate the barrier V_b to electron flow from the metal to the n-type silicon. [10%]

(iii) Based on thermionic emission theory, the current density from the metal to the n-type silicon is given by

$$J_{m-s} = -AT^2 \exp\left(\frac{-eV_b}{kT}\right)$$

where A is the modified Richardson constant and T is the temperature. Give an expression for the current density from the n-type silicon to the metal. [10%]

(c) An insulating oxide layer is inserted between the metal and n-type silicon.

(i) Draw a band diagram of this structure without any external bias applied. Comment on the change in carrier concentration that occurs at the interface of the n-type silicon to the oxide. [15%]

(ii) Give an expression for the threshold voltage V_T that needs to be applied to reach strong inversion at the interface of the silicon to the oxide. Assume that the oxide layer is ideal and contains no fixed or mobile charges. [15%]

4 (a) An abrupt p-n junction is formed from GaAs with a concentration of acceptors in the p-region of $N_A = 10^{24}$ m^{-3}, and a concentration of donors in the n-region of $N_D = 10^{20}$ m^{-3}. Starting from the Poisson equation, show that the width w of the depletion region in the n-type region may be derived to be

$$w = \left(\frac{2\varepsilon_0 \varepsilon_r V_0}{eN_D}\right)^{\frac{1}{2}}$$

where ε_r is the relative permittivity and V_0 the built-in potential. [30%]

(b) This depletion causes a junction capacitance per unit area C_d. Indicate the polarity and calculate the relative value of external bias required with respect to V_0 in order to double C_d. [20%]

(c) The p-n junction is forward biased. The total current density through the diode is constant and given by the sum of the electron and hole current densities J_n and J_p, respectively. Sketch J_n and J_p as function of distance from the depletion region on both sides of the junction. [15%]

(d) In order to suppress electron injection, for the p-type region of the diode the GaAs is replaced by AlGaAs with the same doping density. Draw a band diagram of such a heterojunction under forward bias. [15%]

(e) Explain why a heterojunction between heavily doped AlGaAs and intrinsic GaAs can enable very fast switching times when used as channel for a Metal Semiconductor Field Effect Transistor (MESFET). Comment on whether the AlGaAs should be doped n-type or p-type and draw a band diagram of the doped AlGaAs–GaAs–metal gate structure. [20%]

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