EGT2
ENGINEERING TRIPOS PART IIA

Thursday 30 April 2015 9.30 to 11

Module 3C5

DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A light rigid cubic frame of side $2a$ is shown in Fig. 1. It carries eight masses A-H, each of mass m, at each of its eight vertices. A set of axes x, y, z has its origin at the centre of the cube and is aligned parallel to its edges.

(a) Find the moment of inertia of the cube about the body diagonal AG. [20%]

(b) Masses A at (a, a, a) and G at $(-a, -a, -a)$ are removed from the frame. Find the principal moments of inertia of the cube about its centre. [20%]

(c) A further mass H at $(-a, a, -a)$ is removed so that the total mass of the frame is now $5m$.

(i) Find the inertia matrix of the cube about the origin in the x, y, z frame. Verify that all of its entries are non-zero and equal to either $10ma^2$, $3ma^2$ or ma^2. [30%]

(ii) Verify that one of the principal moments of inertia is $7ma^2$ and find the direction of the corresponding principal axis. [30%]

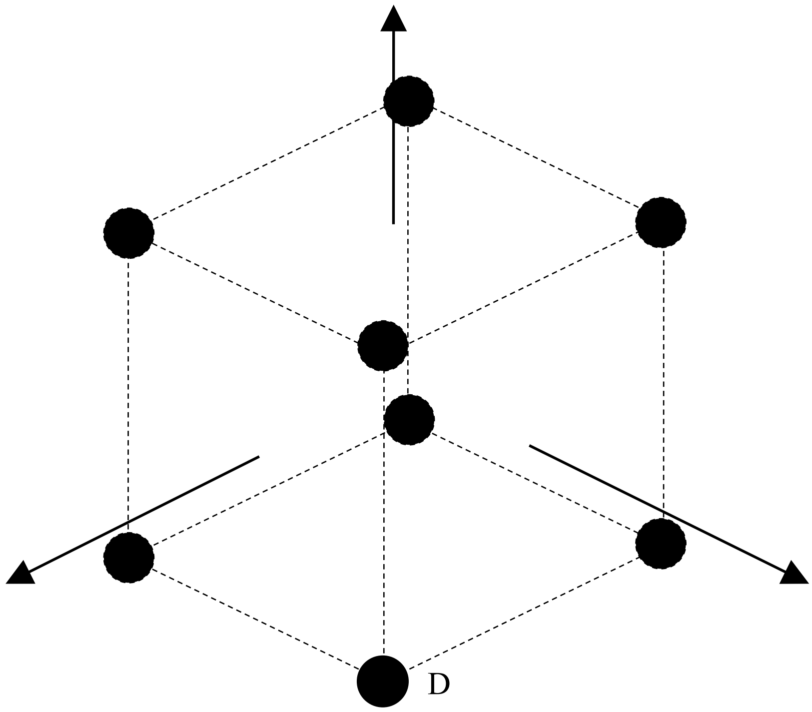

Fig. 1

2 Two thin discs of mass m and radius a are held rigidly together with light links of length a to form an axisymmetric rigid body of mass $2m$ as shown in Fig. 2. The body is rolling without slip on a rigid horizontal table. The angle between the axis of symmetry and the vertical is θ. A steady-state wobbling motion of the body occurs when θ is small and with the centre of mass G motionless.

(a) Find the principal moments of inertia of the body at its centre of mass. [10%]

(b) Show that, for small θ, the magnitude of the steady couple acting on the body about G is approximately $2mga$. [10%]

(c) Show that the rate of wobbling tends to $\sqrt{\frac{2g}{a\theta}}$ as θ tends to zero. [50%]

(d) A spot S is painted on the upper disc and the body is observed to be turning slowly as viewed from above. Find this slow rate of turning for small θ. [30%]

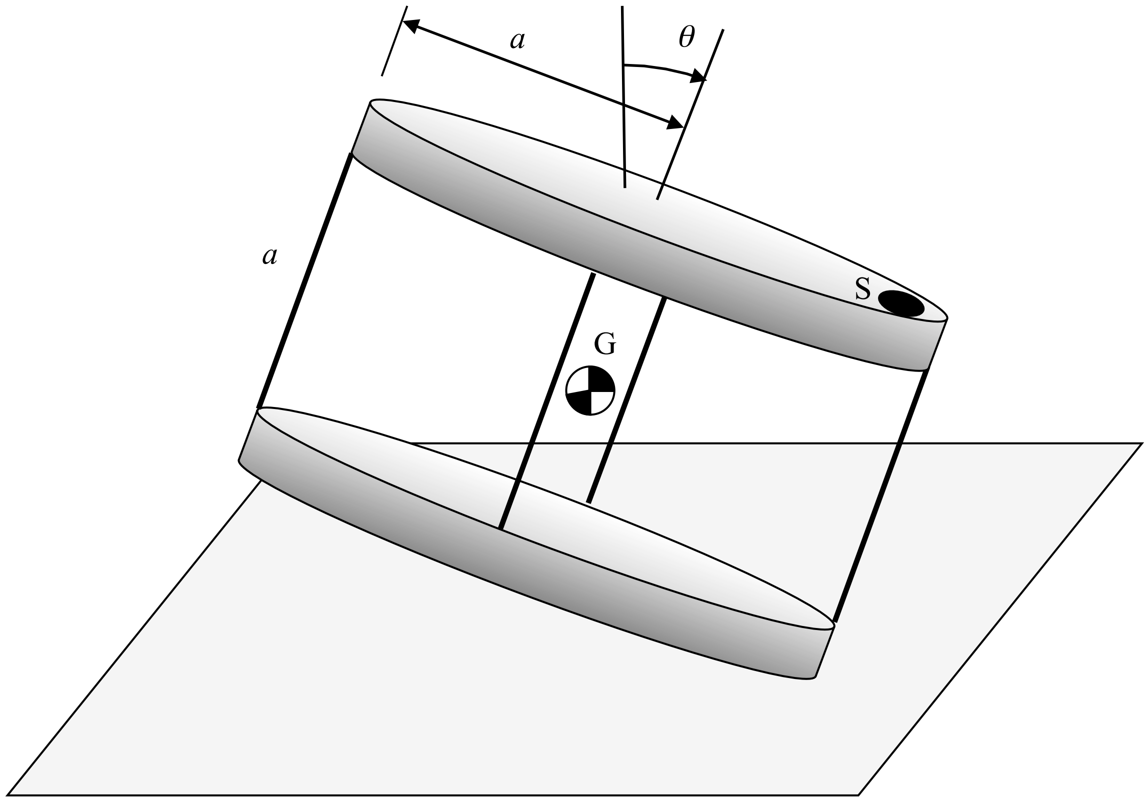

Fig. 2

3 (a) (i) Derive from first principles Euler's equations governing the angular motion of a rigid body whose principal moments of inertia are A, B and C. Use the notation defined on the Data Sheet. [20%]

(ii) Show that a rigid body with $A<B<C$ will be unstable if spinning freely about the axis aligned with B. [30%]

(b) The kinetic energy of a single degree of freedom non-linear system has the form

$$T = A(q)\dot{q}^2,$$

where q is the generalized coordinate and $A(q)$ is a function of q. The potential energy of the system is $V(q)$ and a generalized force Q is applied to the system.

(i) Derive the equation of motion of the system by using Lagrange's equation. [15%]

(ii) Derive an expression for the power input to the system by the generalized force. [10%]

(iii) Show from your result for the power input that if Q=0 then the total energy of the system is conserved. [25%]

4 A schematic of an elastic pendulum is shown in Fig. 3. The system consists of a mass M on a light (approximately massless) rod of unstretched length L, which is free to rotate around the point O in a vertical plane. The stiffness of the rod is k, so that the tension in the rod is given by kx, where x is the extension of the rod. The angle of the rod to the vertical is θ. The mass is subjected to a horizontal applied force F and the acceleration due to gravity is g.

(a) By using Lagrange's equation derive the equations of motion of the system in terms of the generalised coordinates x and θ. [40%]

(b) The system is now placed in the horizontal plane, so that the effect of gravity is removed, and the applied force F is set to zero.

(i) Under free motion, is the generalised momentum conserved for either of the generalised coordinates? Give reasons for your answer. [20%]

(ii) A torque is applied to the system so that the rod is forced to rotate in the horizontal plane at a constant rate $\dot{\theta} = \Omega$. Find the natural frequency of the system and explain what happens if $\Omega^2 > k / M$. [40%]

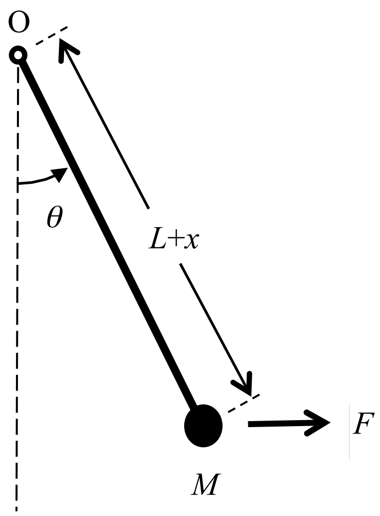

Fig. 3

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