Version HEMH/4

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 4 May 2017 2 to 3.30

Module 3C5

DYNAMICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A thin square plate has sides of length $2a$ and total mass m as shown in Fig. 1(a). The plate lies in the **i**-**k** plane as shown with side AB aligned with **k** and origin O at the midpoint of AB. Find the inertia matrix at O in the **i**-**j**-**k** frame. [20%]

(b) The plate is now rotated by angle θ about **k** as shown in Fig. 1(b). Find the new inertia matrix at O in the **i**-**j**-**k** frame. [30%]

(c) The tail of an arrow of mass m is modelled as three square plates each of mass $m/3$ and side $2a$ fixed at a common edge as shown in Fig. 1(c). The plates are spaced at equal angles. Point O lies on the midpoint of the common edge. Explain why you expect the arrow tail to be an "*AAC*" body and find the inertia matrix at O. [30%]

(d) Find the inertia matrix for an arrow tail comprising N such plates, equispaced and of total mass m . [20%]

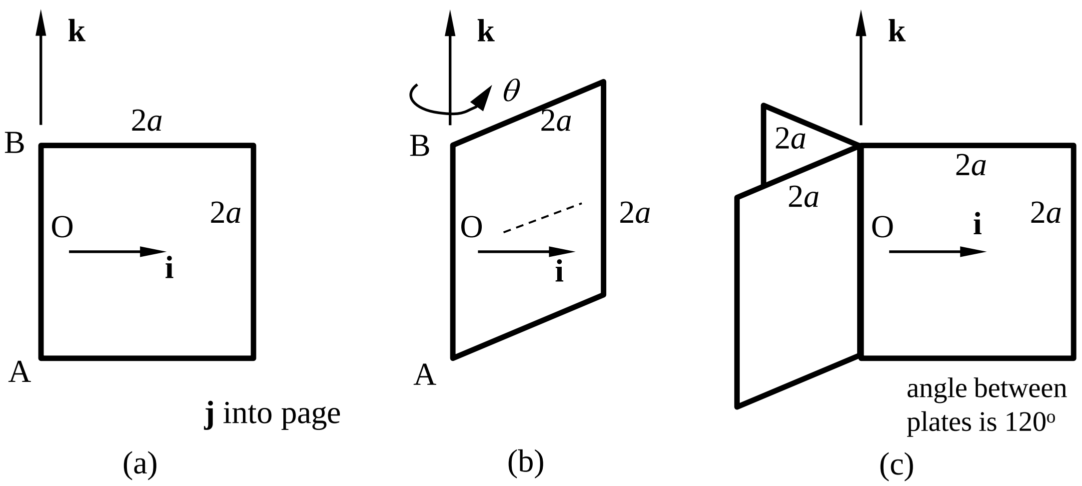

Fig. 1

2 (a) A rigid conical shell has mass m, slanting surface OP of length $2a$ and base of radius a as shown in Fig. 2. Using information in Section 5.6.4 of the Mechanics Data Book, show that the centre of mass is distance $2\sqrt{3}a/3$ from O and find the principal moments of inertia A, A and C at O. [25%]

(b) The cone lies on a rigid horizontal plane and rolls without slip steadily at angular velocity Ω around the vertical axis through O. Use a no-slip condition to show that the angular velocity of the body is $\omega_3 = -3\Omega/2$. [25%]

(c) If the cone is on the verge of tipping the normal reaction R is concentrated at P on the base as shown in the figure. The rolling rate at tipping is $\Omega = \Omega_{max}$. Use the Gyroscope Equations to obtain an expression for the value of Ω_{max}. [40%]

(d) Show that the minimum value of the coefficient of friction required to prevent slipping at the point of tipping is $16\sqrt{3}/39$. [10%]

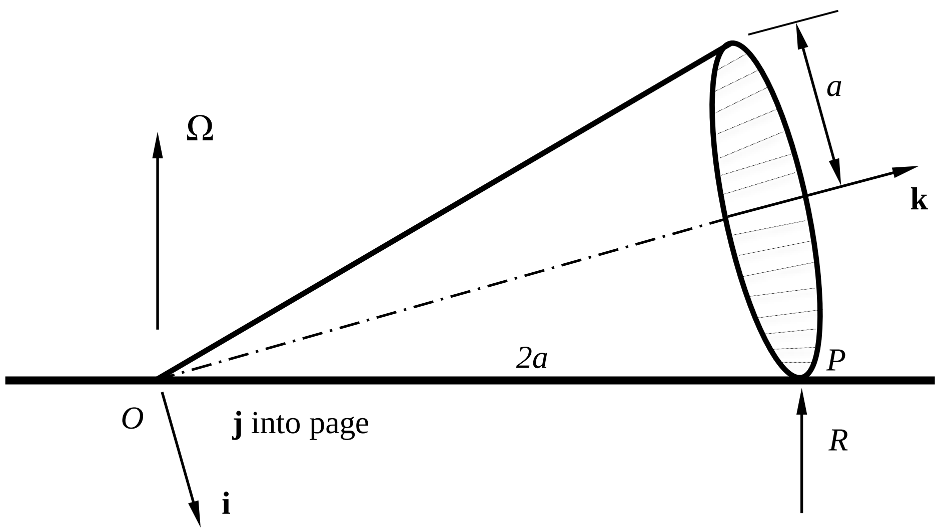

Fig. 2

3 A top of mass m is spinning on a table and the point O in contact with the table is fixed as shown in Fig. 3. Its centre of mass G lies a distance a along the axis from O. It has principal moments of inertia A, A and C about O. The position of the top is described by the usual Euler angles θ, ϕ and ψ as shown in the figure.

(a) In terms of the Euler angles, write down expressions for the angular velocities of the reference frame and of the body. [20%]

(b) Use Lagrange's equations to find the three equations of motion

$$A\ddot{\theta} - A\dot{\phi}^2 \sin\theta\cos\theta + h\dot{\phi}\sin\theta - mga\sin\theta = 0$$

$$C(\dot{\phi}\cos\theta + \dot{\psi}) = h$$

$$A\sin^2\theta\dot{\phi} + h\cos\theta = H$$

and give meanings to the constants h and H. [50%]

(c) For the case of steady precession, find an expression for the precession rate $\dot{\phi}$. Find approximate solutions to this equation for the case of "fast" spin. [30%]

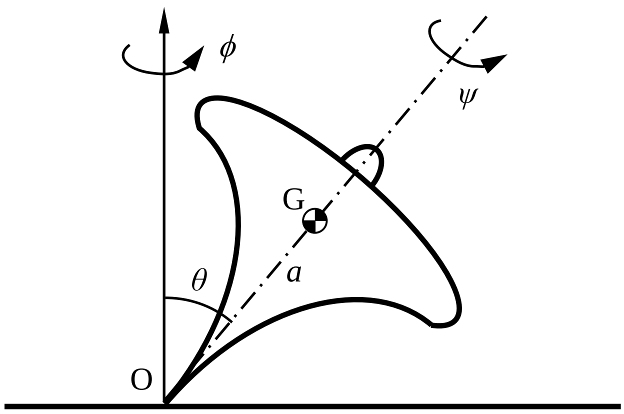

Fig. 3

4 A light rod OA of length a is pivoted at O and constrained to swing through angle θ in a vertical plane as shown in Fig. 4. Perpendicular to OA is a second light rod AB of length b with a point mass m at B. AB is free to rotate by angle ψ about OA. When $\theta = \psi = 0$ the rod OA is hanging vertically and the rod AB is parallel to the pivot at O.

(a) Show that the mass m has velocity components in three orthogonal directions

$$\left(a\dot{\theta} + b\dot{\psi}\cos\psi \,,\, b\dot{\theta}\sin\psi \,,\, b\dot{\psi}\sin\psi\right).$$ [25%]

(b) By using Lagrange's equation, including the effect of gravity, derive the equation of motion associated with the degree of freedom θ. [30%]

(c) The equation of motion associated with the degree of freedom ψ is as follows (you are not required to prove this):

$$b\ddot{\psi} + a\ddot{\theta}\cos\psi - b\dot{\theta}^2\sin\psi\cos\psi + g\cos\psi\sin\theta = 0.$$

Demonstrate that $\psi = \pi/2$ and $\tan\theta = -b/a$ is a static equilibrium point. [15%]

(d) By considering the second derivatives of the kinetic and potential energies, derive the matrix equation that governs small amplitude motions around the static equilibrium position in (c). Hence find the natural frequencies of small vibration around this point. Explain your result in physical terms. [30%]

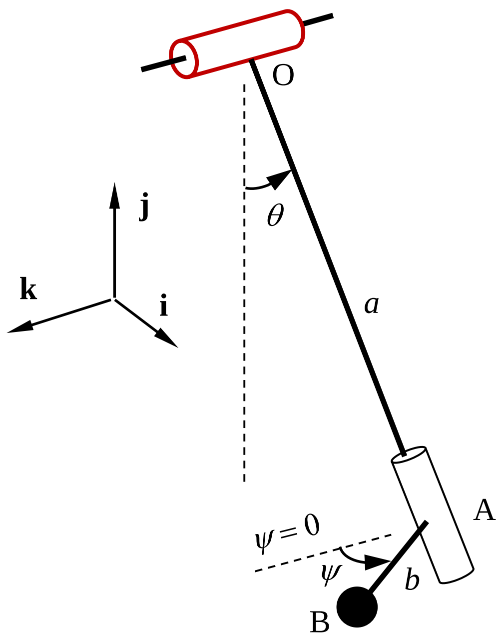

Fig. 4

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