Version JW/3

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 23 April 2015 9.30 to 11

Module 3C6

VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A circular shaft of length L and radius a is made of material with density ρ and shear modulus G. One end of the shaft is prevented from rotating, while the other end is free. The shaft can undergo small-amplitude torsional oscillations. Write down the appropriate governing equation and the boundary conditions at the two ends. Hence find the mode shapes and natural frequencies of the shaft. Sketch the first three mode shapes. [35%]

(b) A shaft of length $2L$ but otherwise identical to the shaft from part (a) has both ends free. Without detailed calculation, sketch the first four mode shapes of this shaft for small-amplitude torsional vibration. [15%]

(c) Equal and opposite torques Q are applied to the two ends of the shaft from part (b) so that it is held in equilibrium, then at time $t = 0$ the torques are suddenly released and the shaft begins to vibrate. Explain why the calculation from part (a) is relevant to this problem, and hence obtain an expression for the subsequent transient vibration in the form of a modal superposition. [50%]

2 (a) A beam of uniform cross-section with second moment of area I has length L and is made of material with density ρ and Young's modulus E. The beam is freely pinned to rigid supports at both ends, and undergoes small-amplitude transverse vibration. Write down the governing equation and appropriate boundary conditions, and hence find the mode shapes and natural frequencies of the beam. [35%]

(b) Explain briefly why the modes of continuous systems must be orthogonal, given that the result has been proved for any discrete system. For the pinned-pinned beam of part (a), show by direct integration that this orthogonality condition is satisfied. [20%]

(c) Making use of appropriate sketches of mode shapes, give a careful plot of the displacement response of the beam of part (a), when driven with a harmonic force at distance $L/4$ from the left-hand end and observed at the point $L/3$ from the same end. Use a decibel scale, and include a frequency range that covers the first 12 natural frequencies. Label and explain important features. Assume small damping. You may find it helpful to use $\omega^{1/2}$ rather than ω for the horizontal axis. [45%]

3 Three uniform rods, each with mass m and length L, are joined together with frictionless hinges and supported in a plane as shown in Fig. 1. The two outer ends are pinned to simple supports and there are two springs of stiffness k located at the hinges as shown. A small mass εm, where $\varepsilon << 1$, is attached to the inner rod at position x. The displacements y_1 and y_2 describe small deflections from static equilibrium. Neglect the effects of gravity and assume that tension in the rods and bending of the rods can be neglected.

(a) For the case where $\varepsilon = 0$, derive an equation for the kinetic energy of the system and show that the mass matrix is

$$M = m\begin{bmatrix} 2/3 & 1/6 \\ 1/6 & 2/3 \end{bmatrix}$$ [20%]

(b) For the case where $\varepsilon = 0$, sketch the natural mode shapes of vibration of the system and calculate the natural frequencies. [20%]

(c) For the case where $\varepsilon \neq 0$, use Rayleigh's principle to show that the lower natural frequency becomes

$$\omega^2 \approx \frac{k}{m}\left(\frac{6}{5+3\varepsilon}\right)$$

and find a corresponding approximate expression for the higher natural frequency. [40%]

(d) What position of the additional mass makes the difference between the two natural frequencies (i) largest; and (ii) smallest? Justify your answers. [20%]

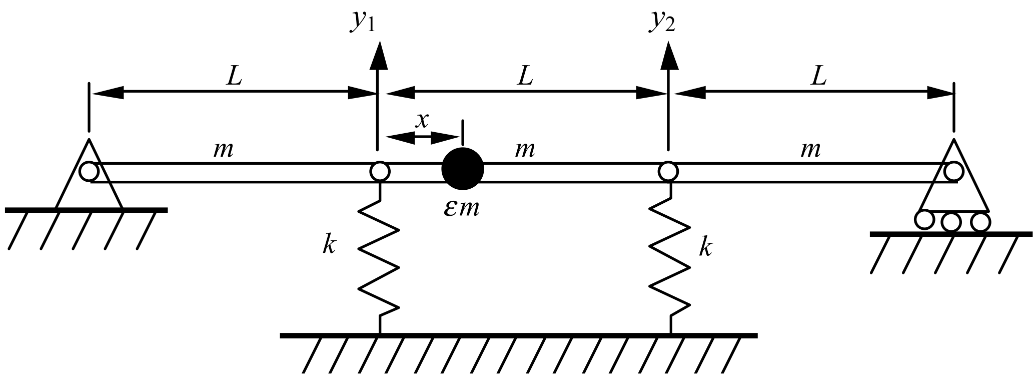

Fig. 1

4 A five-element discrete model of a satellite, used to calculate axial vibration transmission, is shown in Fig. 2. Each element has mass m and is joined to adjacent elements with springs of stiffness k.

(a) Without calculating the eigenvalues or eigenvectors, sketch plausible mode shapes for each of the five natural frequencies of the discrete model. [20%]

(b) Explain why two of the natural modes have eigenvectors that can be expressed in the form $[-1\ \ -\alpha\ \ 0\ \ \alpha\ \ 1]^T$. Use Rayleigh's method to estimate the lower of the two corresponding natural frequencies. [20%]

(c) By differentiating Rayleigh's quotient, or otherwise, calculate the exact values of α and the natural frequencies of the modes in part (b). Which modes are these? [30%]

(d) The base of the satellite is excited by an axial sinusoidal force $F \sin \omega t$. Sketch a graph of the magnitude of the displacement y_4 (on a dB scale) as a function of angular frequency ω. [20%]

(e) The base of the satellite is now forced to vibrate with a vertical *displacement* of ±1 mm at a frequency corresponding to the lowest non-zero natural frequency. What is the amplitude and phase of the displacement at the top of the satellite? [10%]

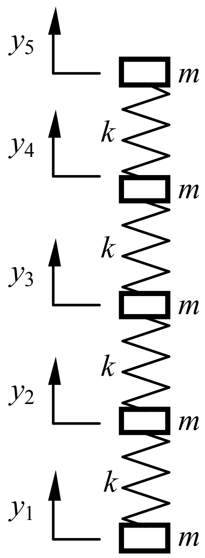

Fig. 2

END OF PAPER

Answers

1(a) Modes $u_n(x) = \sin\frac{(n-1/2)\pi x}{L}$, $\omega_n = \frac{(n-1/2)\pi c}{L}$, $c = \sqrt{G/\rho}$ for n = 1,2,3...

(c) Step response $\frac{4LQ}{Ga^4\pi^3}\sum_n \frac{(-1)^{n+1}}{(n-1/2)^2}\sin\frac{(n-1/2)\pi x}{L}\cos\frac{(n-1/2)\pi ct}{L}$

2(a) Modes $u_n(x) = \sin\frac{n\pi x}{L}$, $\omega_n = \left(\frac{n\pi}{L}\right)^2\sqrt{\frac{EI}{\rho A}}$ for n = 1,2,3...

3(b) Modes $[1,1]$ with $\omega_1^2 = \frac{6k}{5m}$, $[1,-1]$ with $\omega_2^2 = \frac{2k}{m}$

(c) Higher frequency $\omega_2^2 \approx \frac{2k}{m\left[1+\varepsilon(1-2x/L)^2\right]}$

4(c) $\alpha = \frac{-1 \pm \sqrt{5}}{2}$, $\omega_n^2 = \frac{3 \mp \sqrt{5}}{2}\frac{k}{m}$