EGT2
ENGINEERING TRIPOS PART IIA

Friday 22 April 2016 9.30 to 11

Module 3C6

VIBRATION

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Supplementary page: one extra copy of Figs. 2 and 3 (Question 3)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) The in-plane vibration of a three-bladed propeller may be modelled in idealised form as shown in Fig. 1. A rigid circular disc of radius a and polar moment of inertia I is freely pinned at its centre. It carries three identical, equally-spaced cantilevers, each consisting of a mass m on a leaf spring, such that the masses when in equilibrium lie at a radius b. The circumferential displacements of the masses are denoted respectively by x_1, x_2 and x_3 relative to the positions they would occupy if the springs were undeformed. In terms of these displacements, the springs all have stiffness k. The angle of rotation of the hub is denoted θ. Find the mass and stiffness matrices in terms of the vector of generalised coordinates $[\theta\ x_1\ x_2\ x_3]^T$. Ignore any effects of centrifugal stiffening. [20%]

(b) Which vibration modes can be written down immediately, taking advantage of the symmetry of the system? Verify them, and find the corresponding natural frequencies. Find the remaining mode(s) by taking advantage of orthogonality, or otherwise, and find each corresponding natural frequency. [60%]

(c) Because of manufacturing tolerances, one spring is found to have a different stiffness $k' > k$. Which of the modes found in part (b) are unaffected? Without detailed calculation, what can be said about the others? Sketch the likely new mode shapes. [20%]

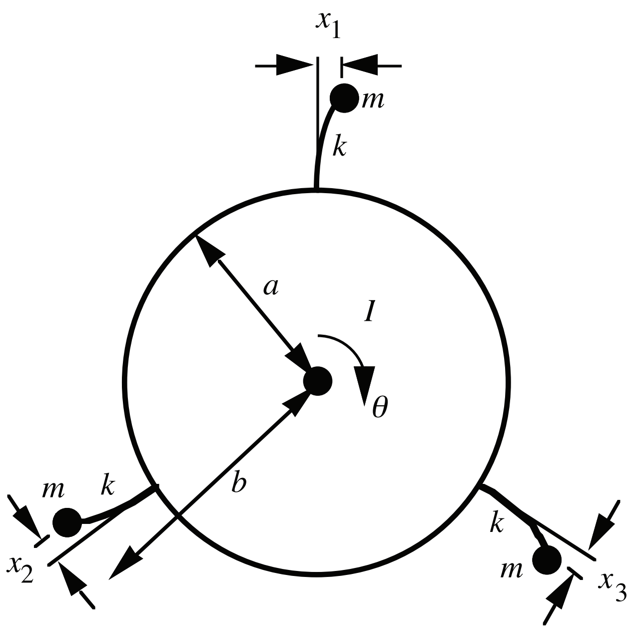

Fig. 1

2 (a) Axial vibration of an oil-well drillstring can be modelled as follows. A suspended bar of length L and uniform cross-section with area A, made of a material of density ρ and Young's Modulus E, can undergo small amplitude axial oscillations. The top of the bar is fixed, while a harmonic axial force of magnitude F and frequency ω is applied at the bottom of the bar. Any effects of tension in the bar due to its self-weight can be ignored. Write down the appropriate governing equation and the boundary conditions at the two ends. [20%]

(b) Find a closed-form expression for the transfer function $G(L,z,\omega)$ from the input force F to the axial displacement y, where z is the depth (measured downwards from the fixed top end) at which the response is observed. [35%]

(c) For the particular case of the driving point response (i.e. $z = L$), find the poles and zeros of the transfer function. What is the physical meaning of the zeros? [15%]

(d) Axial vibration of the drillstring is excited by a tri-cone drill bit, which causes a sinusoidal axial *displacement* of magnitude Y_0 and frequency ω to be imposed to the bottom of the drillstring. The excitation frequency ω is related to the angular velocity of the drillstring Ω_0 by $\omega = 3\Omega_0$. During drilling, there is an average compressive load W applied to the bottom of the drillstring. Using the expression for the transfer function $G(L,L,\omega)$ from part (b), derive a condition to guarantee that the drillbit does not lose contact with the rock. Estimate angular velocities that should be avoided to ensure smooth drilling. [30%]

3 (a) Simulated measurements of frequency response functions of two vibrating systems are shown in Figs. 2 and 3. The measured output variable was velocity, and the response function magnitude has been plotted on a dB scale. For both cases, discuss what can be deduced about the nature of the system, the boundary conditions, and the positions of the driving and measurement points. *An additional copy of Figs. 2 and 3 is attached to the back of this paper. It may be detached, annotated to highlight significant features and handed in with your answers.* [80%]

(b) If these had been real measurements rather than simulations, what features of the plots would you expect to be different? What steps might be taken when performing the measurements to minimise unwanted additional features? [20%]

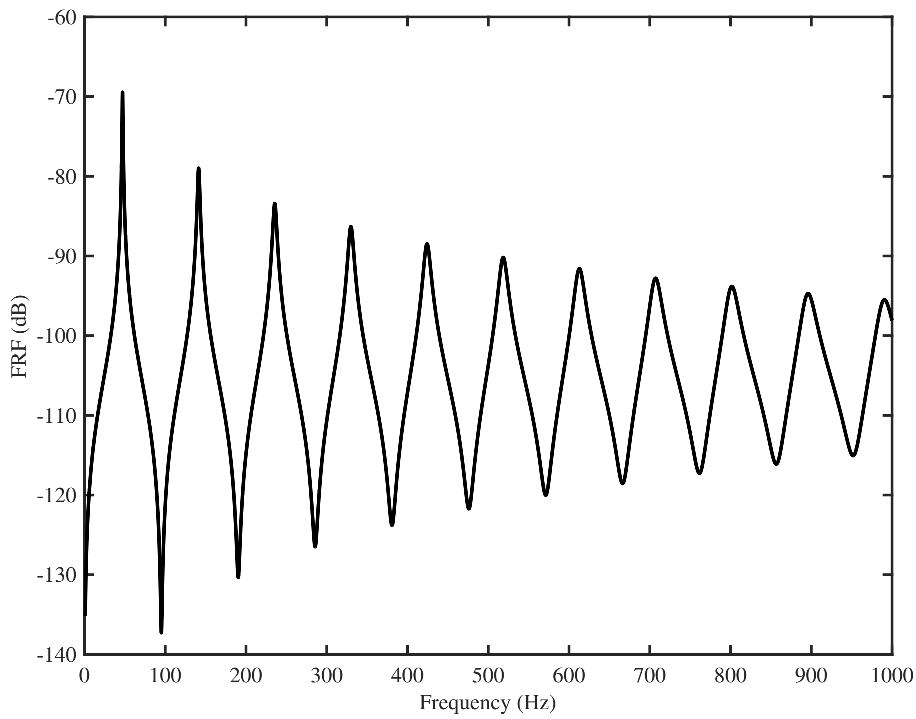

Fig. 2

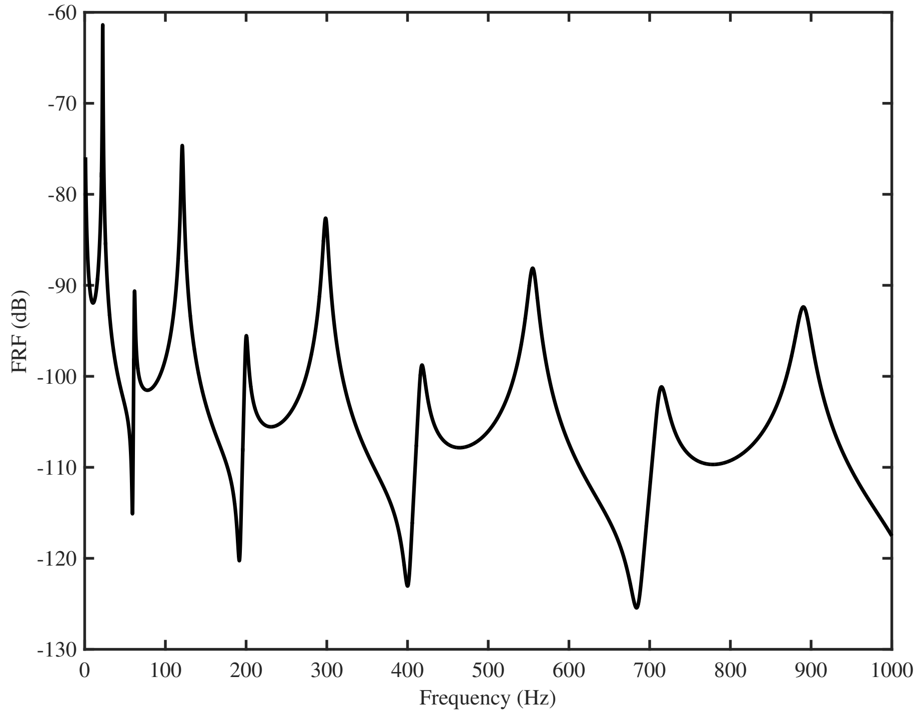

Fig. 3

(TURN OVER

4 (a) An electronics board is mounted on a structure by clamping it at one end and leaving the other end free. The board can be modelled as a cantilever beam of length L, cross-sectional area A and second moment of area I. It is made from a material of Young's modulus E and density ρ.

(i) Write down the equation governing small transverse vibration, and derive the dispersion equation governing the propagation of harmonic waves on the beam. [10%]

(ii) Find an expression whose roots give the natural frequencies of the beam. [30%]

(iii) Using a suitable sketch, explain why a good estimate can be made for all modes except the first, and derive an expression for these frequencies. [20%]

(b) Assuming a mode shape of the form $U(x) = x^2$, use Rayleigh's principle to estimate the first natural frequency. [30%]

(c) The normalised mode shapes for the beam all have approximately equal displacement U_L at the free end. Using this approximation, find an expression for the transient response of the beam when an impulsive force is applied at the free end. [10%]

END OF PAPER

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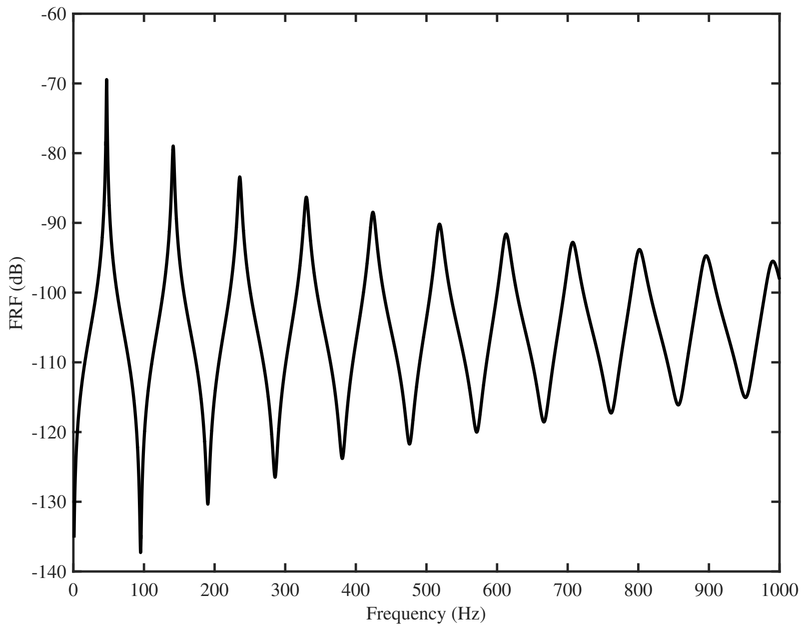
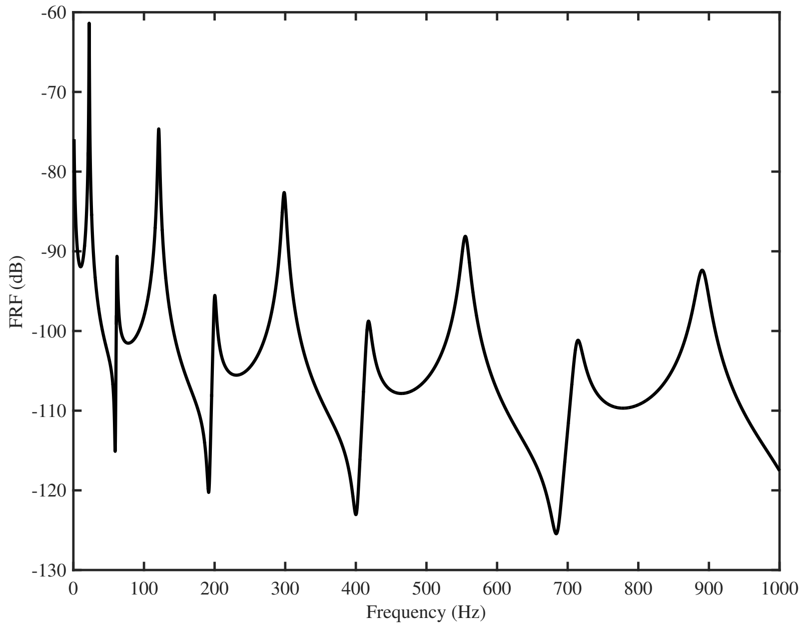
ENGINEERING TRIPOS PART IIA

Friday 22 April 2016, Module 3C6, Question 3.





Extra copy of Figs. 2 and 3 for Question 3.

Answers

1(a)

$$K = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & k & 0 & 0 \\ 0 & 0 & k & 0 \\ 0 & 0 & 0 & k \end{bmatrix}, \quad M = \begin{bmatrix} I+3mb^2 & mb & mb & mb \\ mb & m & 0 & 0 \\ mb & 0 & m & 0 \\ mb & 0 & 0 & m \end{bmatrix}$$

(b)

$$\mathbf{u}_1 = [1\ 0\ 0\ 0]^t, \omega_1 = 0$$

$$\mathbf{u}_2 = [0\ 0\ 1\ \ -1]^t, \omega_2 = \sqrt{k/m}$$

$$\mathbf{u}_3 = [0\ 1\ 0\ \ -1]^t, \omega_3 = \sqrt{k/m}$$

$$\mathbf{u}_4 = [1\ A\ A\ A]^t, A = b + \frac{I}{3mb}, \omega_4 = \sqrt{\frac{k(I+3mb^2)}{mI}}$$

For modes 2 and 3, any other linear combinations of these two are also possible.

2(b)

$$G = \frac{c\ \sin(\omega z/c)}{EA\omega\cos(\omega L/c)}$$

(c) Poles at $\dfrac{\omega L}{c} = (n - \dfrac{1}{2})\pi, n = 1, 2, 3$

Zeros at $\dfrac{\omega L}{c} = n\pi, n = 1, 2, 3$

(d) For smooth drilling require $\dfrac{3Y_0 EA\Omega_0}{c\tan(3\Omega_0 L/c)} < W$

4(a)(i) Dispersion relation $\omega = \sqrt{\dfrac{EI}{\rho A}}k^2$

(a) (ii) $\cos kL \cosh kL = -1$

(b) $\omega \approx \sqrt{\dfrac{20EI}{\rho A L^4}}$