EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 27 April 2017 9.30 to 11

Module 3C6

VIBRATION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C5 Dynamics and 3C6 Vibration data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) A stretched string of length L has tension P and mass per unit length m. Both ends of the string are fixed, the distance from one end of the string is x and small amplitude transverse deflection of the string is denoted y.

(i) By solving the partial differential equation governing the free small amplitude vibration of the string, derive an expression for the mode shapes and natural frequencies of the string. [20%]

(ii) Sketch the first three mode shapes of the string. [10%]

(iii) The string is driven at $x = x_1$ by a sinusoidal force of amplitude F at frequency ω. Using your result from (a)(i) find an expression for the transfer function $G_a(x_1, x_2, \omega)$ from the input force F to the output transverse displacement y measured at an arbitrary position x_2 along the string. [20%]

(b) Two strings of the same length L and mass per unit length m are connected at $x = L/3$ from each of their ends by a light, rigid strut, as illustrated in Fig. 1. The tensions of the two strings are $P_1 = P_2 = P$.

(i) What is the driving point transfer function G_b at the connection point of the coupled system? [10%]

(ii) Sketch the magnitude of G_b on a log-amplitude scale, including the first three peaks of the coupled system. [10%]

(iii) Sketch the first six mode shapes of the coupled system. Identify any mode shapes that correspond to peaks from your sketch of the transfer function G_b. [30%]

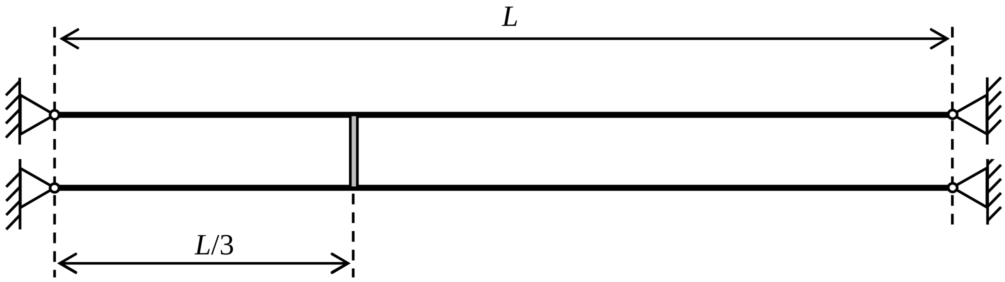

Fig. 1

2 (a) A simple bridge is made using a uniform beam of length L, pinned at each end. The bridge has a solid rectangular cross section of width b and thickness d, Young's Modulus E and density ρ.

(i) Derive an expression for the mode shapes and natural frequencies of the beam. [20%]

(ii) What would be the effect on the mode shapes and natural frequencies if the thickness of the whole beam was increased? Justify your answer. [10%]

(b) It is necessary to modify the bridge so that the first natural frequency is higher. It is proposed to add two springs to the structure as illustrated in Fig. 2. The springs are both of stiffness k and are placed a distance αL from each end.

(i) Assuming sinusoidal mode shapes, use Rayleigh's principle to estimate the factor by which the natural frequencies of the beam are increased. [40%]

(ii) For α = 0.1, which natural frequencies would be most affected in terms of their absolute frequency change (i.e. not in terms of their fractional change)? [10%]

(iii) Without detailed calculation, what would happen to the natural frequencies and mode shapes in the limit as $k \to \infty$, with α small but not zero. [20%]

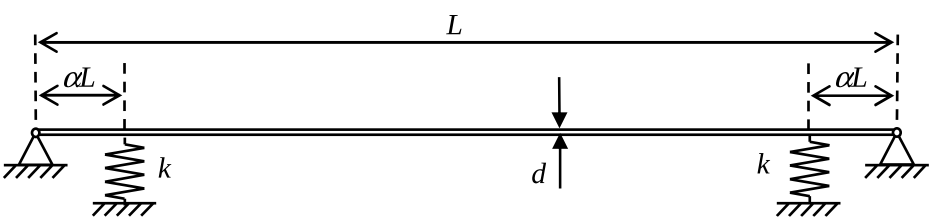

Fig. 2.

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3 Four particles of mass m are attached to a tightly-stretched, light wire as shown in Fig. 3. The wire carries a constant tension P and the effect of gravity is negligible. The length of each segment of the wire is L as shown.

(a) Show that for small lateral displacements of the particles y_1, y_2, y_3, y_4, the potential energy of the system is given by

$$V = \frac{P}{L}\left[y_1^2 + y_2^2 + y_3^2 + y_4^2 - y_1 y_2 - y_2 y_3 - y_3 y_4\right],$$

and write down the kinetic energy. [20%]

(b) Sketch the natural mode shapes for small lateral vibration. [30%]

(c) One of the natural modes has the form $[1\ \alpha\ \alpha\ 1]^{\mathrm{T}}$, where α is a constant. By differentiating Rayleigh's Quotient, determine α and the corresponding natural frequency. Determine the natural frequencies and mode shapes for any other modes that can be calculated using this same form of mode shape. [30%]

(d) A sinusoidal force $x(t) = X\sin\omega t$ is applied to the top mass, as shown. Sketch a graph of the amplitude of the displacement response of the second mass y_2 as a function of frequency ω. Use a dB scale. [20%]

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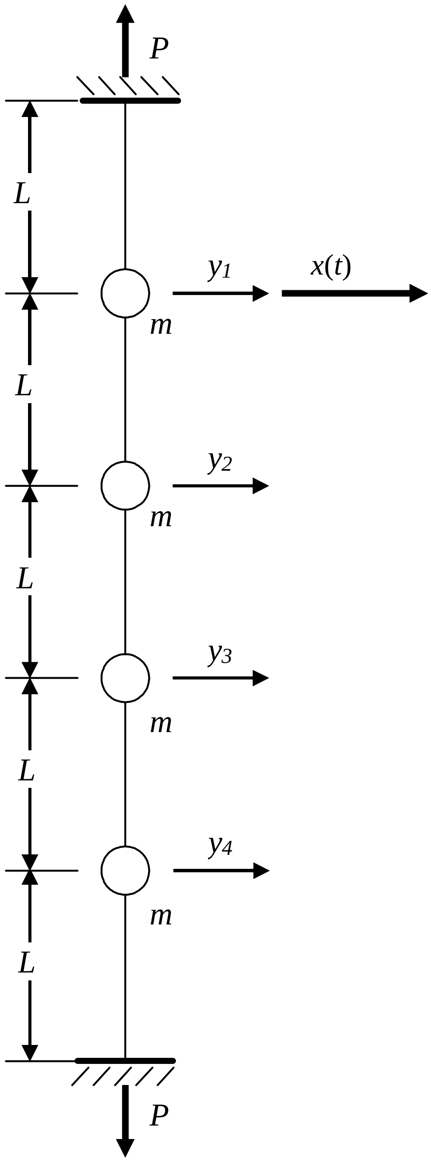

Fig. 3.

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4. The twin axle 'walking beam' suspension of a heavy lorry can be modelled as shown in side view in Fig. 4. A light, rigid beam of length $2a$ joins two axles of mass m. The beam is connected in the middle to an undamped spring of stiffness k through a frictionless pivot. This spring is attached to the body of the vehicle, which has mass $4m$. The axles have undamped tyres of stiffness $2k$, which are effectively connected to the ground. The centre of the beam has vertical displacement y_1, the vehicle body is constrained to move vertically with displacement y_2, and the beam has pitch rotation θ.

(a) Write expressions for the kinetic energy and potential energy for small vibrations of the suspension and show that the stiffness matrix can be written as

$$\begin{bmatrix} 5k & -k & 0 \\ -k & k & 0 \\ 0 & 0 & 4a^2k \end{bmatrix}$$ [20%]

(b) Calculate the natural frequencies. Calculate and sketch the natural mode shapes. [40%]

(c) Use Rayleigh's Quotient to estimate the change of mass of the vehicle body needed to reduce the natural frequency of the lowest vibration mode by 10%. [30%]

(d) Suggest design changes that would introduce damping into the suspension. Comment on the damping of each mode. [10%]

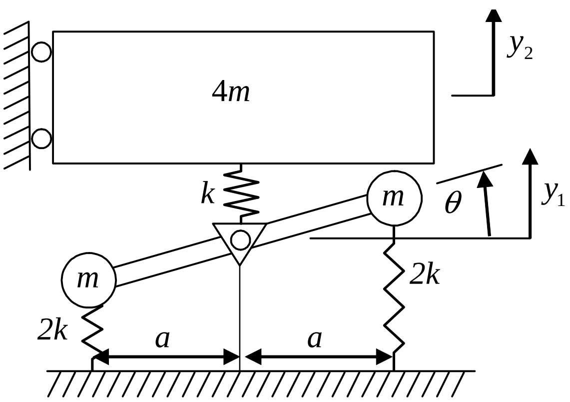

Fig. 4

END OF PAPER

Answers

1. (a) (i) $U_n = A\sin\dfrac{n\pi x}{L}$, $\omega_n = \dfrac{n\pi}{L}\sqrt{\dfrac{P}{m}}$ (iii) $G_a(x_1, x_2, \omega) = \dfrac{2}{mL}\sum_{n=1}^{\infty}\dfrac{\left(\sin\dfrac{n\pi x_1}{L}\right)\left(\sin\dfrac{n\pi x_2}{L}\right)}{\omega_n^2 - \omega^2}$

(b) (i) $G_b = \dfrac{1}{mL}\sum_{n=1}^{\infty}\dfrac{\left(\sin\dfrac{n\pi}{3}\right)^2}{\omega_n^2 - \omega^2}$

2. (a) (i) $U_n = A\sin\dfrac{n\pi x}{L}$, $\omega_n = \left(\dfrac{n\pi}{L}\right)^2\sqrt{\dfrac{EI}{\rho A}}$ (ii) No change to U_n, increase in ω_n

(b) (i) $1 + \dfrac{4kL^3(\sin n\pi\alpha)^2}{EI(n\pi)^4}$ (ii) n = 5, 15, 25,…

(iii) Approaches clamped-clamped case

3. (a) (i) $T = \dfrac{1}{2}m\left(\dot{y}_1^2 + \dot{y}_2^2 + \dot{y}_3^2 + \dot{y}_4^2\right)$

(c) $\alpha = \dfrac{1 \pm \sqrt{5}}{2}$, $\omega_1 = 0.618\sqrt{\dfrac{P}{Lm}}$, $\omega_3 = 1.618\sqrt{\dfrac{P}{Lm}}$

4. (a) $T = \dfrac{1}{2}m\left(2\dot{y}_1^2 + 4\dot{y}_2^2 + 2a^2\dot{\theta}^2\right)$, $V = \dfrac{1}{2}k\left(5y_1^2 + y_2^2 - 2y_1y_2 + 4a^2\theta^2\right)$

(b) $\omega_1^2 = 0.196\dfrac{k}{m}$, $\omega_2^2 = 2\dfrac{k}{m}$, $\omega_3^2 = 2.554\dfrac{k}{m}$

(c) Increase sprung mass by 24% to 4.96m

(d) A damper in parallel with k damps modes 1 and 3, but not mode 2, which is a troublesome undamped pitch mode. Therefore: either add a torsional damper at the pivot or add dampers between the two axles (masses m) and the body.