EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 29 April 2015   9.30 to 11

Module 3C7

MECHANICS OF SOLIDS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C7 formulae sheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 It has been suggested that stresses derived from the Airy Stress Function

$$\phi(x,y) = \frac{\sigma_0}{\alpha^2}\cos(\alpha x)(1+\alpha y)e^{-\alpha y}, \quad \text{where } \alpha = \frac{2\pi}{\lambda}$$

may be suitable to describe the stresses within a half-space defined by $y \geq 0$, subjected to a sinusoidally varying normal load of maximum magnitude σ_0, as shown in Fig. 1. Assume plane stress. The material has a yield strength in pure shear of k.

(a) Calculate the stresses within the half-space, and show that they give the elastic solution to the problem. [40%]

(b) Show that size of the in-plane Mohr's circle does not depend upon x. Assuming a Tresca criterion, find the value of σ_0 to give first yield. [40%]

(c) Assuming a Von Mises criterion, find the value of σ_0 to give first yield. [20%]

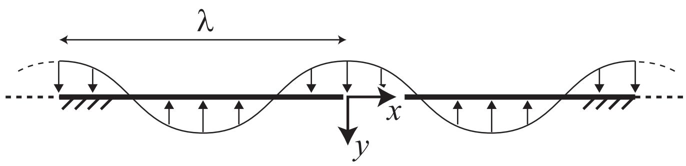


Fig. 1

2 (a) Show that the Airy Stress Function

$$\phi(r,\theta) = M\left(\frac{r}{a}\right)^2 + N\ln\left(\frac{r}{a}\right)$$

is consistent with Lame's solution for the elastic stresses in an axisymmetric system. [20%]

(b) A plate subjected remotely to a uniform biaxial stress contains a small circular hole of radius a. Calculate the stress concentration factor. [30%]

(c) An elastic plate subjected remotely to loading in shear contains a small circular hole. Show that the stresses calculated from the Airy Stress Function given below satisfy the required boundary conditions and hence calculate the stress concentration factor.

$$\phi(r,\theta) = a^2\sigma_0\cos 2\theta\left(-\frac{1}{2}\left(\frac{r}{a}\right)^2 + 1 - \frac{1}{2}\left(\frac{a}{r}\right)^2\right)$$

[50%]

3 (a) In a plane strain axisymmetric linear elastic system, the radial displacement at a radial position r is given by

$$u(r) = Cr + \frac{D}{r},$$

where C and D are constants. Show that this gives rise to stresses that satisfy equilibrium. [20%]

(b) A long cylinder is made of an elastic material with Young Modulus E and Poisson Ratio ν. The cylinder has an inner radius a and outer radius $2a$ and is subjected to an internal pressure such that the inner radius expands by 1%.

(i) If the outer radius is unconstrained, find the expansion of the outer radius. [40%]

(ii) If the outer radius is constrained so that it cannot expand, find the external pressure applied to the cylinder. [40%]

4 A rigid ideally-plastic material with a tensile yield strength σ_Y yields according to the Tresca yield criterion. Circular and square long pipes with a central internal hole of radius a as sketched in Figs. 2a and 2b, respectively are made from this material. The pipe is subjected to an internal pressure p. Assume plane strain conditions with an axisymmetric velocity field with a radial component

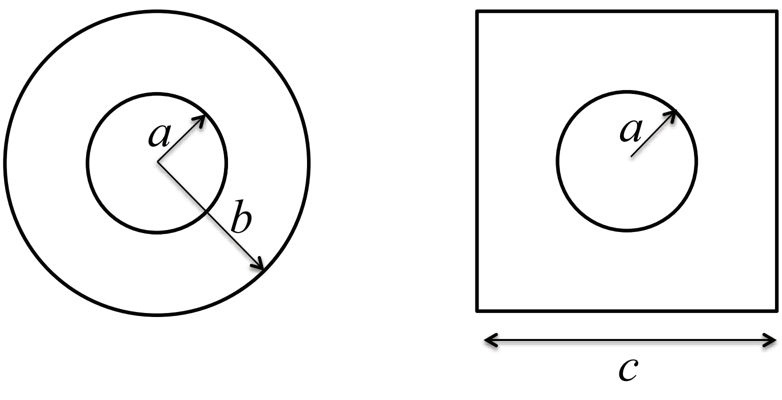
$$u = A/r$$

where A is a constant and r the radial co-ordinate measured from the center of the pipe.

(a) For the circular pipe of outer radius b (Fig. 2a) calculate the upper bound to the pressure p required to burst the pipe. [30%]

(b) For the square pipe of side c (Fig. 2b) calculate the internal plastic dissipation. Hence calculate the upper bound to the pressure p required to burst the pipe. You may assume that $\int_0^{\pi/4} \ln(\cos\theta)\mathrm{d}\theta = -0.086$. [50%]

(c) Discuss qualitatively the accuracy of the collapse pressures obtained for the circular and square pipes. [20%]



(a) (b)

Fig. 2

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Numerical answers

1. (b) $\sigma_0 = ke$

   (c) $\sigma_0 = \frac{\sqrt{3}}{2} ke$

2. (b) SCF = 2

   (c) SCF = 4

3. (b)(i) $u = 0.04a\left(\frac{1-\nu}{5-2\nu}\right) + \frac{0.02a}{(5-2\nu)}$

   (b)(ii) $p = \frac{0.02E}{3}\left[\frac{1-\nu}{(1+\nu)(1-2\nu)}\right]$

4. (a) $p = \sigma_Y \ln{(b/a)}$

   (b) $p = \frac{4\sigma_Y}{\pi}\left[\frac{\pi}{4}\ln\left(\frac{c}{2a}\right) + 0.086\right]$