Version VSD/3

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 25 April 2017   9.30 to 11

Module 3C7

MECHANICS OF SOLIDS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C7 formulae sheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A light uniform cantilever of depth $2b$ and length L is loaded by a uniform normal force per unit length q on the top and bottom surfaces as shown in Fig. 1. A candidate Airy stress function for representing the stress field in this cantilever is

$$\phi = \frac{A}{6}x^2y^3 + \frac{B}{3}x^3y + \frac{C}{2}x^2y + \frac{D}{20}y^5$$

where A, B, C and D are constants.

(a) What are the requirements on the constants for ϕ to be a valid Airy stress function? [20%]

(b) Determine the constants A, B, C and D in terms of q and b such that the stresses determined from ϕ satisfy the boundary condition on $y = \pm b$. Hence provide expressions for the stresses σ_{xx}, σ_{yy} and σ_{xy}. [40%]

(c) Show that the stress field obtained above gives the correct shear force distribution along the length of the cantilever. [20%]

(d) Determine the bending moment at $x = 0$ due to the stress field obtained in (b). Hence modify the stress field obtained in (b) in order to satisfy boundary conditions at the free end of the cantilever. [20%]

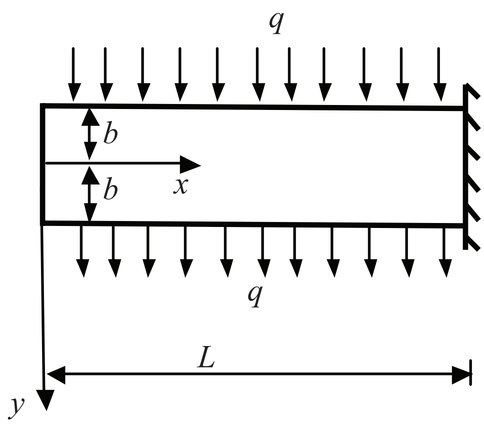

Fig. 1

2 A shaft made from a material with a shear modulus G has an equilateral triangular cross-section as shown in Fig. 2. The shaft is subjected to an axial torque T and twists by an angle α per unit length along its longitudinal axis.

(a) With the co-ordinate system (x, y) defined in Fig. 2, show that a Prandtl stress function given by

$$\phi = -\frac{G\alpha}{2}\left(\eta y^2 - x^2\right)\left(1 - \frac{x}{a}\right)$$

is appropriate to determine the stress state in the shaft. Hence determine the constant η. [20%]

(b) Calculate the stiffness T/α of the shaft. [30%]

(c) Determine the location of the maximum shear stress and its magnitude. [30%]

(d) Giving reasons, specify all locations on the cross-section of the shaft where the shear stress vanishes. [20%]

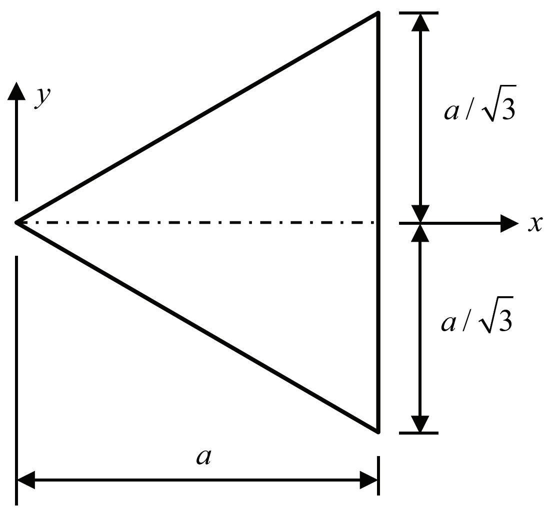

Fig. 2

3 (a) The von Mises-Schleicher yield criterion is given by

$$f([\sigma]) = \bar{\sigma}^2 + (c_1 - c_2)I_1 - c_1c_2 = 0$$

where $\bar{\sigma}$ is the von Mises effective stress, I_1 is the first stress invariant, and c_1 and c_2 are material constants.

(i) Explain why this criterion has the built-in assumptions of isotropic pressure-sensitive behaviour. [20%]

(ii) Determine c_1 and c_2 in terms of the uniaxial tensile yield strength σ_t and uniaxial compressive yield strength σ_c. [20%]

(b) Figure 3 shows a long thick-walled cylinder with both ends closed that has an inner radius a and outer radius b. It is made from a solid of Young's modulus E and shear yield strength k, and the Tresca yield criterion is assumed. The outer surface is subjected to a pressure p. Considering a cross-section A-A answer the following questions.

(i) Derive the equilibrium equation relating $\sigma_{\theta\theta}$ and σ_{rr}. [15%]

(ii) Determine the stresses σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} for an elastic response. [25%]

(iii) Determine the pressure p_e when yield is initiated and the pressure p_u when the entire cross-section is plastic. Assume that $\sigma_{\theta\theta} < \sigma_{zz} < \sigma_{rr}$. [20%]

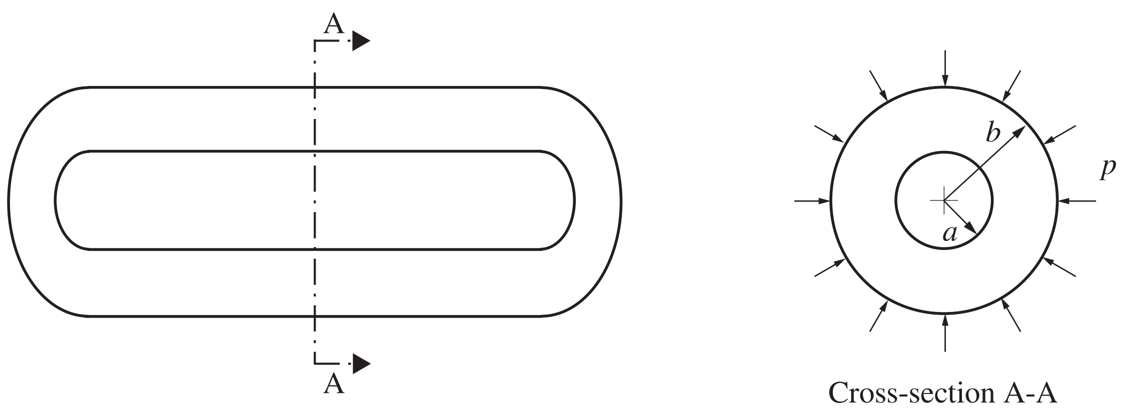

Fig. 3

4 (a) Sketch the von Mises and Tresca yield surfaces in the π-plane. [15%]

(b) Explain the difference between the upper bound and lower bound theorems of plasticity. [20%]

(c) Figure 4 shows a plane strain extrusion process in which a work-piece is driven at a speed $v = 1$ into the die with width $2H$ and exits with a height $2h$. The material of the work-piece is assumed to be rigid-perfectly plastic with a flow stress in shear k. Tangential velocity discontinuities occur along the three thin lines indicated in the figure.

(i) Assume that the interfaces between the die and the deforming material are well lubricated. Derive first an upper bound expression for the extrusion pressure p in terms of the angle α and then determine the minimum value of p. [55%]

(ii) In an equivalent drawing process the workpiece is pulled from the right by a drawing stress t (and the extrusion pressure is $p = 0$). Determine the drawing stress t. [10%]

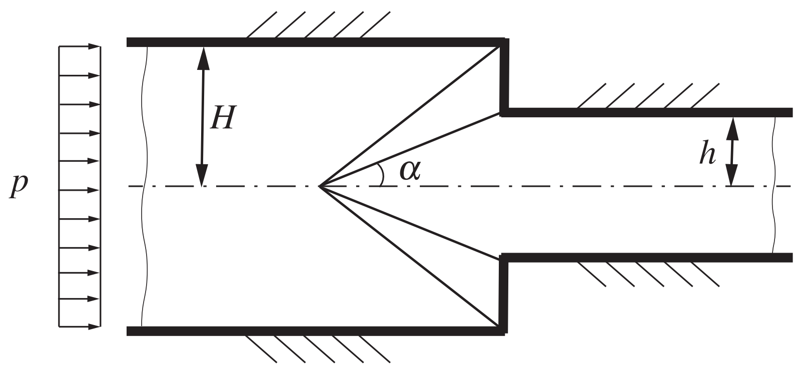

Fig. 4

END OF PAPER

THIS PAGE IS BLANK