EGT2
ENGINEERING TRIPOS PART IIA

Friday 24 April 2015 2 to 3.30

Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Summarise briefly the conditions which must be satisfied in order that a contact can be described by the classical Hertzian equations. [10%]

(b) Figure 1a shows a long rigid cylinder of radius R loaded against an elastic half-space by a normal radial load P. The material of the half-space has Young's modulus E and Poisson's ratio ν. The contact patch extends from $x=-b$ to $x=+b$ and the pressure distribution $p(x)$ can be assumed semi-elliptical:

$$p(x)=p_0\left(1-\frac{x^2}{b^2}\right)^{1/2} . \qquad (1)$$

(i) If $b \ll R$ use the expressions for b and p_0 in the Data Sheet to find an expression for the peak Hertz pressure p_0 in terms of the load per unit length P' and b. [10%]

(ii) The cylinder rolls in the positive x direction slowly and steadily under the action of a horizontal force F. If it is assumed that the pressure between $x = 0$ and $x = -b$ is zero and the pressure between $x = 0$ and $x = +b$ is increased but otherwise the distribution remains as described by equation (1), show that

$$F=\frac{4}{3\pi}\frac{Pb}{R} .$$ [50%]

(iii) An experiment to investigate rolling friction using a steel roller and a rubber half-space generates the data in the table below and plotted in Fig. 1b. To what extent are these in accord with the relationship in (ii)?

normal force P (N)	9.81	19.6	39.2	68.7	85.3	98.1	113
horizontal force F (N)	0.18	0.36	0.98	1.86	2.84	4.22	6.47

[20%]

(iv) Suggest how this model of rolling friction might be improved. [10%]

Hint for part (ii): $\sin^3\theta = \frac{3}{4}\sin\theta-\frac{1}{4}\sin 3\theta$

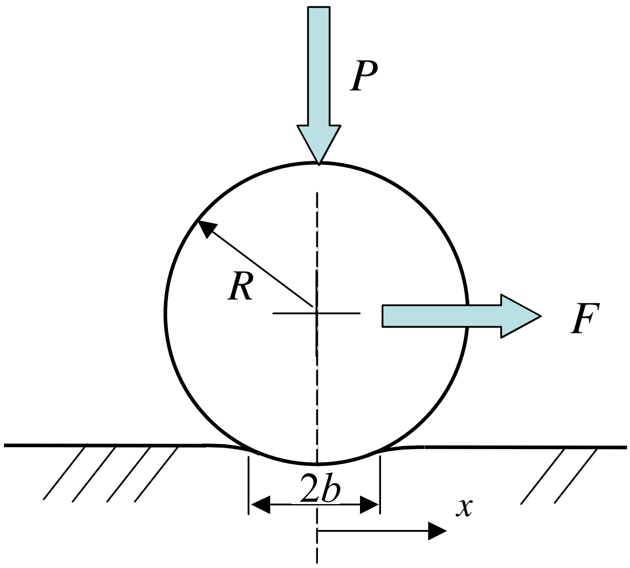
P
R
F
2b
x

Fig. 1a

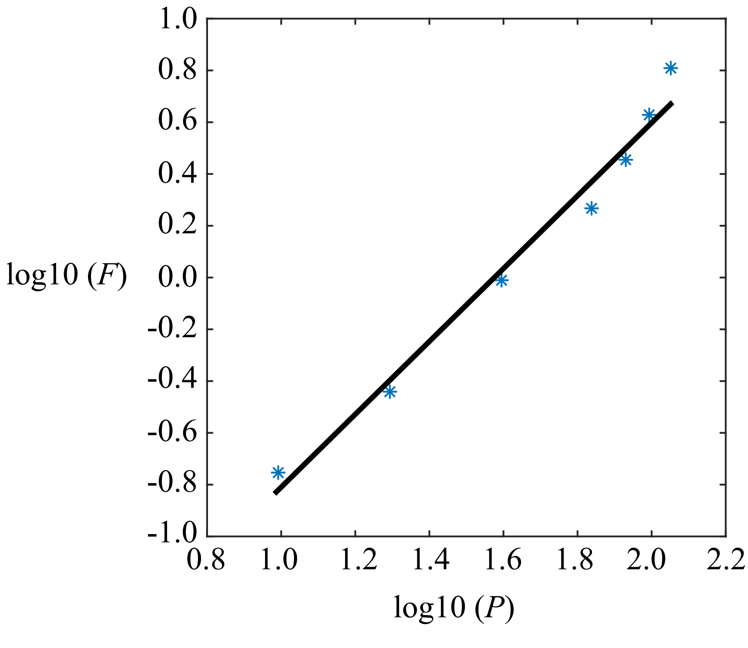
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
log10 (F)
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
log10 (P)

Fig. 1b

2 The output characteristic of a bicycle rider working at constant effort is shown in Fig. 2, where the mean torque T measured at the crank depends linearly on the angular speed ω of the crank. The intercepts on the axes are T_0 = 30 N m and ω_0 = 20 rad s^{-1}. The bicycle and rider have combined mass 100 kg and the wheel radius R = 0.33 m. The ratio of rear wheel speed Ω to crank speed ω is given by $G = \Omega / \omega$. The ratio can be varied by the rider. The force resistance F (in N) to motion on level ground is given by $F = 0.4V^2$, where V (in m s^{-1}) is the speed of the bicycle.

(a) (i) Calculate the maximum power output of the rider and the corresponding crank speed. [15%]

(ii) Find the maximum speed of the bicycle on level ground, and find the corresponding speed ratio G. [25%]

(b) An electric motor directly drives the front wheel of the bicycle. To conform to UK regulations on Electrically Assisted Pedal Cycles, the motor has a maximum power output of 200 W and provides no assistance when the speed of the bicycle is above 7 m s^{-1}. The maximum output torque of the motor is 13.33 N m.

(i) If the speed ratio G = 3 and the rider and motor are both providing power, sketch a graph of maximum acceleration as a function of speed from 0 m s^{-1} to 10 m s^{-1}. Mark on your graph values of acceleration and speed at discontinuities in the gradient of the graph. [50%]

(ii) It is proposed to redesign the bicycle so that an electric motor drives the crank instead of the front wheel. Briefly discuss the significant design and performance issues associated with this change. [10%]

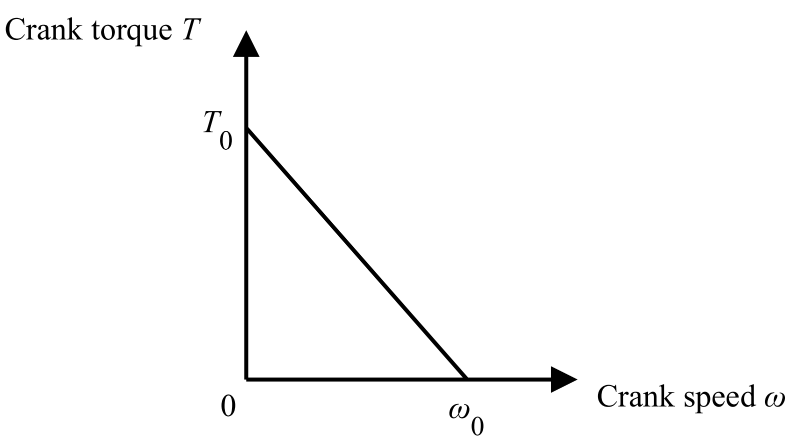

Fig. 2

3 (a) Derive the epicyclic speed rule given on the 3C8 data sheet:

$$\omega_s = (1+R)\omega_c - R\omega_a$$

and define the symbols in this expression. [15%]

Figure 3 illustrates a compound epicyclic gearbox with tooth numbers $A_1 = 80$, $A_2 = 90$, $S_1 = 20$ and $S_2 = 30$. Planet carrier C_1 is fixed to ground. The input speed is 300 revolutions per minute and the input power is 2 MW. Power losses in the gearbox may be neglected.

(b) For the case where A_2 is fixed:

(i) show that the ratio of input to output speeds is 1:16, with a reversal of direction; [20%]

(ii) find the restraint torque on annulus A_2. In what sense does this torque act, relative to the direction of the input torque? [45%]

(c) For the case where A_2 is coupled to A_1, so that both rotate at the same speed in the same direction, find the ratio of input to output speeds. [20%]

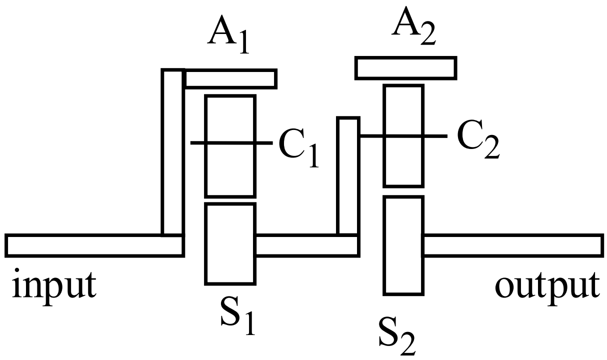

Fig. 3

4 A pair of standard spur gears is required to transmit power of 50 kW. The shaft speed is 2000 revolutions per minute, the speed ratio is unity, and the distance between shaft centres is exactly 100 mm. The pressure angle is 20° and the addendum is equal to the module. The allowable contact stress is 1 GPa and the allowable bending stress is 300 MPa. The contact modulus E^* is 115 GPa.

(a) (i) Determine the minimum tooth width if the allowable contact stress is not to be exceeded for single tooth contact at the pitch point. [20%]

(ii) Determine the minimum module if the allowable bending stress is not to be exceeded. Assume that the tooth width is the value calculated in (i), and that the gears are made imperfectly. [20%]

(b) The shaft of each gear wheel is supported by two deep groove ball bearings, type 6008, located symmetrically either side of the gear wheel.

(i) Calculate the life of a bearing in hours, assuming 2% probability of failure. Assume that oil of the correct viscosity is used. [20%]

(ii) Briefly discuss the design issues associated with supporting a shaft with two deep groove ball bearings and suggest a possible mounting arrangement for the bearings. [10%]

(c) The shaft of each gear wheel is now required to support an axial load of up to 1 kN in either direction. The deep groove ball bearings are replaced by two taper roller bearings, type 32008, located symmetrically either side of the gear wheel. The bearings are preloaded to ensure that no axial clearance arises. The induced axial force F_a due to applied radial force F_r is given by $F_a = F_r / (2Y)$ where Y=1.6. Determine the required axial preload force and the corresponding preload displacement if the axial stiffness of each bearing is 10 MN m^{-1}. [30%]

END OF PAPER

Answers

1 (b) (i) $p_0 = \dfrac{2P'}{b\pi}$

2 (a) (i) 150 W, 10 rad s^{-1}

(ii) 7.21 m s^{-1} , 2.185

(b) (i) 0.7 m s^{-2} at 0 m s^{-1}

0.53 m s^{-2} at 5 m s^{-1}

0.29 m s^{-2} at 7 m s^{-1}

0.0 m s^{-2} at 7 m s^{-1}

−0.25 m s^{-2} at 10 m s^{-1}

3 (b) (ii) 3e5/(8π) N m = 11.94 kN m, same direction

(c) − 1/19

4 (a) (i) 21.8 mm

(ii) 4 mm

(b) (i) 795 hours

(c) 1294 N, 0.256 mm

D J Cole