Version DJC/4

EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 19 April 2016 2 to 3.30

Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).
Supplementary page: one extra copy of Fig. 1 (Question 1)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The output characteristic (maximum torque T versus speed ω) of an electric motor is shown in Fig. 1. The torque is a constant 200 N m between 0 and 500 rad s^{-1}. Between 500 rad s^{-1} and 1500 rad s^{-1} the power output is constant. The maximum speed of the motor is 1500 rad s^{-1}.

The motor provides power to a road vehicle with total mass m = 1000 kg and wheel radius R = 0.3 m. The resistance F (in N) to forward motion on level ground is given by $F = 227 + 0.4V^2$, where V (in m s^{-1}) is the vehicle speed. The ratio of motor speed ω to wheel speed Ω is given by $G = \omega / \Omega$.

An additional copy of Fig. 1 is attached to the back of this paper. It should be detached and handed in with your answers.

(a) Determine the maximum speed of the vehicle on level ground for $G = 5$. [30%]

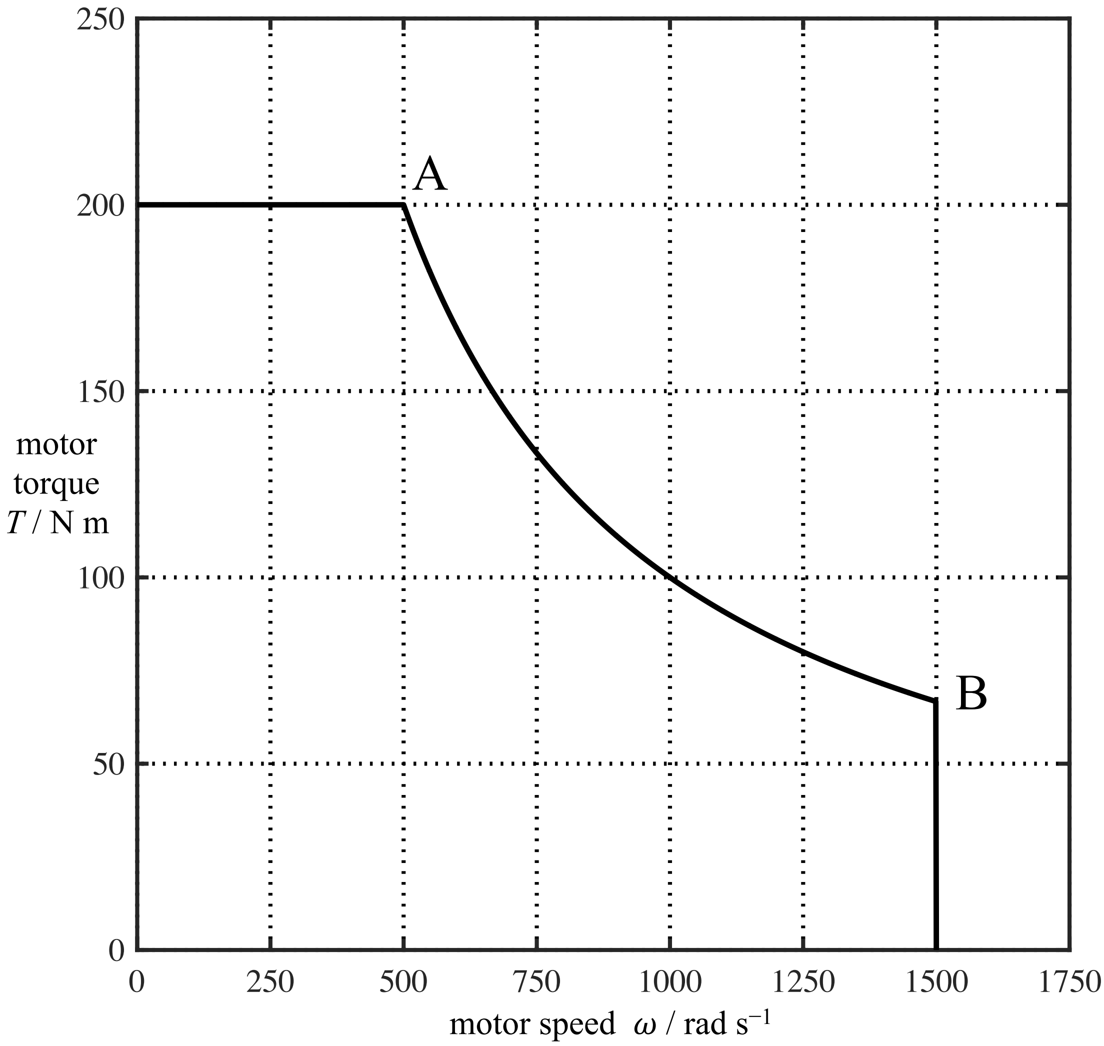
(b) Sketch a graph of maximum vehicle acceleration $\mathrm{d}V/\mathrm{d}t$ (in m s^{-2}) versus vehicle speed V (in m s^{-1}) for $G = 5$. Indicate salient values on your graph. [30%]

(c) Different values of G could put the operating point at A or at B, shown on Fig. 1. Comment on the behaviour of the vehicle at each of these operating points. [15%]

(d) Acceleration performance is improved by providing two ratios: $G = 15$ and $G = 5$.

(i) For $G = 15$ determine the vehicle speed and maximum vehicle acceleration when the motor speed is 1500 rad s^{-1}. Compare with the acceleration available for $G = 5$ at the same vehicle speed. [15%]

(ii) Without further detailed calculation, sketch a graph of maximum vehicle acceleration versus vehicle speed, indicating the speed range over which each ratio should be used. [10%]



250
200
150
100
50
0
motor
torque
T / N m
A
B
0
250
500
750
1000
1250
1500
1750
motor speed ω / rad s⁻¹

Fig. 1

2 Figure 2 illustrates an $n \times n$ stack of long elastic cylindrical rods of diameter d, length $L = nd$ (out of the plane of the figure) and plane strain Young's modulus $E' = E/(1-\upsilon^2)$. The stack is reduced in height by an amount Δ as it is compressed between flat rigid plates with an average pressure p. The rods are held in a regular square array and just touch each other in the unloaded state. The loaded contacts at the top and bottom of each rod can be described using Hertz contact theory.

(a) For a single contact between two rods, show that the maximum Hertzian contact pressure p_0 is given by:

$$p_0 = \left(\frac{2pE'}{\pi} \right)^{1/2} .$$ [20%]

(b) Derive an expression for the total contact area between each layer of rods, in terms of E', p, n and d. [15%]

(c) Figure 3 illustrates an alternative configuration, where alternate layers of rods are orientated at 90° to each other. For this configuration derive expressions for:

(i) the maximum Hertzian contact pressure at the rod-to-rod contacts;

(ii) the total contact area between each layer.

Comment on the difference between these results and those of parts (a) and (b). [45%]

(d) For the configuration of Fig. 3, derive an expression for the compliance Δ/p of the stack, assuming that all the elastic deformation is associated with deformation at the rod-to-rod contacts and neglecting deformation at the contacts with the rigid plates. [20%]

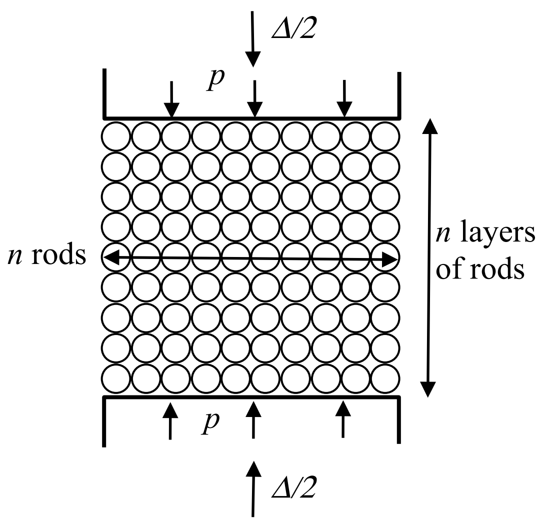


Fig. 2

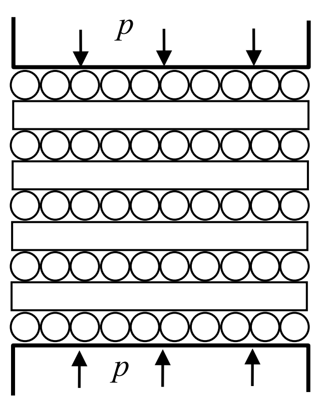


Fig. 3

3 The planetary gear system of a vehicle transmission consists of a sun wheel, an annulus, and a carrier with three planet wheels. The wheels and annulus are spur gears with standard involute geometry. The sun wheel has 24 teeth and the annulus has 96 teeth. The pitch circle diameter of the annulus is 192 mm. The sun wheel rotates at 5000 revolutions per minute (rpm) and the carrier rotates at 2000 rpm in the opposite direction to the sun wheel. The power input to the sun wheel is 50 kW. There are no losses.

(a) Calculate the ratio of sun wheel torque to annulus torque, and the ratio of carrier torque to annulus torque. [25%]

(b) Calculate the magnitude and direction of flow of power at the carrier and at the annulus. . [20%]

(c) Each planet wheel is attached to the carrier by a rolling element bearing located well within the dedendum circle of the planet wheel. If the bearing is to have a life of 1000 hours with 1% probability of failure, determine the dynamic load rating *C* required of a roller bearing. [55%]

4 (a) Explain the meaning of 'balanced design' in relation to spur gears. Explain why a balanced design might not be desirable. For a given diameter of gears state how the module affects the balance of the design. [10%]

(b) A pair of precisely manufactured spur gears has the following specification:

number of pinion teeth	N_1	31		
number of wheel teeth	N_2	60		
module	m	3	mm	
addendum	a	3	mm	
path of contact	l_1	7.35	mm	(using data sheet equation)
path of contact	l_2	7.90	mm	(using data sheet equation)
face width	w	30	mm	
pressure angle	ϕ	20	degrees	
contact modulus	E^*	115	GN m^{-2}.	

(i) Find the smallest effective radius of curvature for single contact and for double contact (note that l_1 and l_2 have been calculated for you). [30%]

(ii) Find the maximum pinion torque if the maximum allowable contact stress is 1200 MN m^{-2}. [30%]

(iii) Find the maximum pinion torque if the maximum allowable bending stress is 300 MN m^{-2}. [20%]

(iv) Comment on the balance of the design. [10%]

END OF PAPER

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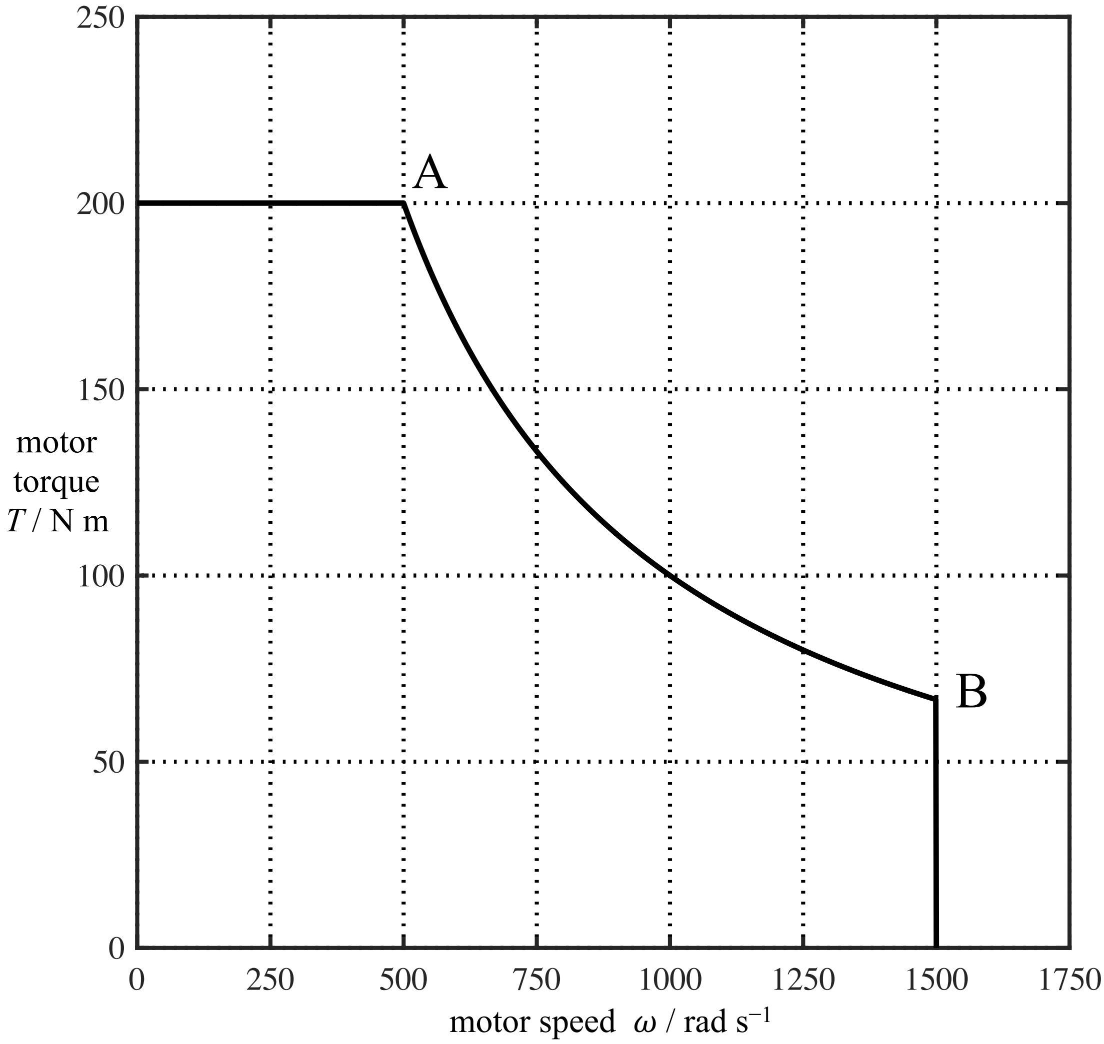
EGT2 Candidate Number:

ENGINEERING TRIPOS PART IIA

Tuesday 19 April 2016, Module 3C8, Question 1.



Extra copy of Fig. 1: Motor output characteristic for Question 1.

Answers

1 (a) 60 m/s

1 (d) (i) 30 m/s, 2.8 m/s^2 (for G=5 and G=15)

1 (d) (ii) use G=15 from 0 m/s to 30 m/s then G=5 from 30 m/s to 60 m/s

2 (b) $2n^2d^2\sqrt{\frac{2p}{\pi E'}}$

2 (c) (i) $\frac{1}{\pi}(6pE'^2)^{\frac{1}{3}}$ (ii) $n^2d^2\pi\left(\frac{3}{4}\frac{p}{E'}\right)^{\frac{2}{3}}$

2 (d) $\frac{(n-1)}{2}36^{\frac{1}{3}}\frac{d}{E'^{\frac{2}{3}}p^{\frac{1}{3}}}$

3 (a) 0.25, −1.25

3 (b) 100 kW into carrier, 150 kW out of annulus

3 (c) 23 kN

4 (b) (i) 9.96 mm (single contact), 6.63 mm (double contact)

4 (b) (ii) 514 Nm

4 (b) (iii) 490 Nm