Version MPFS/4

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 27 April 2017 14.00 to 15:30

Module 3C8

MACHINE DESIGN

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 3C8 data sheet (9 pages).
Supplementary page: one extra copy of Fig. 2 (Question 1)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The output characteristic of a bicycle rider working at constant heart rate (effort) is shown in Fig. 1, where the mean torque T applied to the crank by the rider depends linearly on the angular speed ω of the crank. The intercepts on the axes are T_0 = 80 N m and ω_0 = 20 rad s^{-1}.

(a) Briefly describe a laboratory experiment that could be performed to determine the maximum power output of a cyclist working at constant heart rate. [20%]

The ratio of crank speed ω to rear wheel speed Ω is given by $G = \omega / \Omega$. The rider and a bicycle have combined mass m = 100 kg, frontal area A = 0.5 m^2, drag coefficient C_d = 0.813, and wheel radius R = 0.33 m. The force resistance F (in N) to constant speed motion is given by $F = mg\alpha + 0.5\rho A C_d V^2$, where V is the speed of the bicycle in m s^{-1}, α is the gradient of the road, density of air ρ = 1.23 kg m^{-3}, and force due to gravity g = 10 N kg^{-1}. The bicycle is equipped with a freewheel so that energy cannot flow into the rider from the bicycle when the bicycle is travelling forwards. The relationship between F and V when $\alpha = 0$ is shown in Fig. 2; also shown are lines of constant FV. *An additional copy of Fig. 2 is attached at the end of this paper. If used for graphical construction it should be detached and handed in with your answers.*

The rider and bicycle travel a total of 30 km in three stages of 10 km. The first 10 km are travelled at constant speed on zero gradient. The second 10 km are travelled at constant speed on a gradient of 0.1. The final 10 km are travelled at constant speed on a gradient of −0.1. The transient acceleration at the beginning of each stage can be ignored.

(b) Find the speed ratio G_1 required to complete the first stage of the course in minimum time. Find the speed ratios G_2 and G_3 required to complete the second and third stages of the course in minimum time. Hence find the minimum total time to travel the 30 km distance. [35%]

(c) The bicycle develops a fault and remains in a ratio G = 0.66 for all three stages. Find the new minimum total time to travel the 30 km distance. [35%]

(d) Without further calculation and assuming that optimum speed ratios can be used, explain how the rider's effort might be distributed across the three stages to minimise the time required to travel the 30 km distance. [10%]

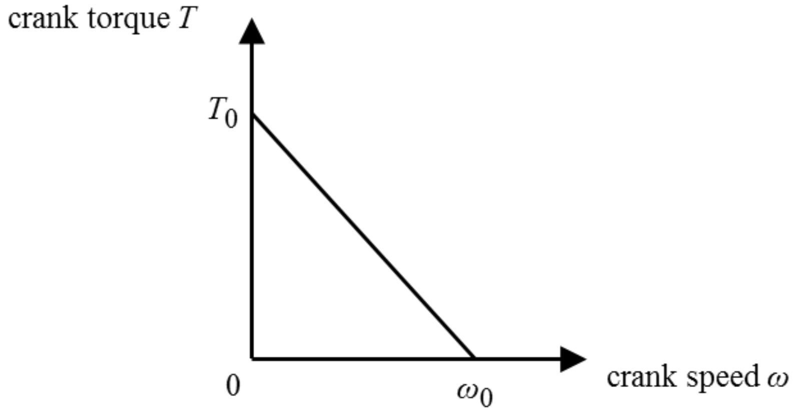

Fig. 1

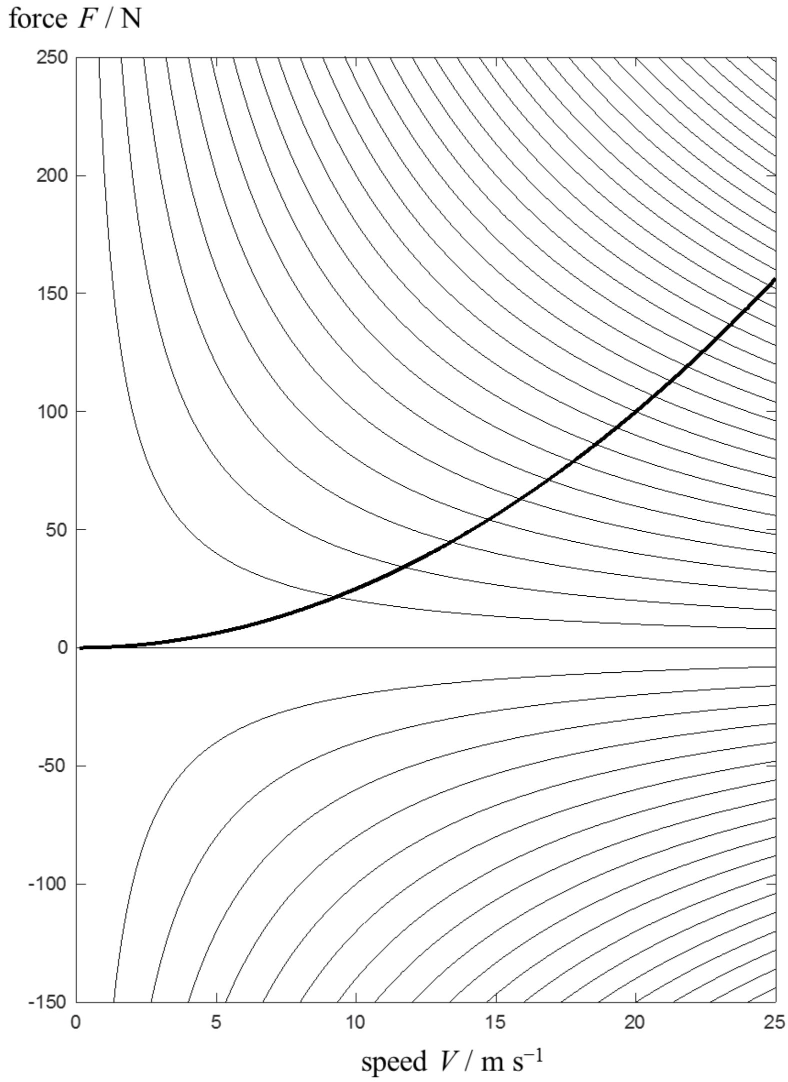

Fig. 2

(TURN OVER

2 Figure 3 shows a cutting tool designed for use in the garden. It consists of three bodies: two handles and a cutting blade (shaded grey). The bodies are connected by frictionless pivots at G and H. The two handles incorporate teeth (not shown) with accurate involute profiles that mesh together. The pitch point is P. The user applies a force F = 100 N to the handles. The cutting blades apply a force W to the branch (not shown) placed between the cutting blades. Friction at the gear tooth contacts may be neglected.

The pressure angle is 20 degrees. The module is 8 mm and the pitch circle radius r is 52 mm. The width of the teeth is 10 mm and the addendum is equal to the module. The Young's modulus of the teeth material is 70 GPa, Poisson's ratio is 0.33 and yield strength is 300 MPa.

(a) Draw free-body diagrams showing the forces acting between the three bodies of the assembly and hence or otherwise find the force W. [20%]

(b) Find the force acting along the pressure line. [5%]

(c) Find the minimum radius of curvature for single contact and for double contact. [45%]

(d) Find the worst case contact stress. How could the design be modified to reduce the contact stress, assuming the same values of F and W are required? [30%]

(cont.

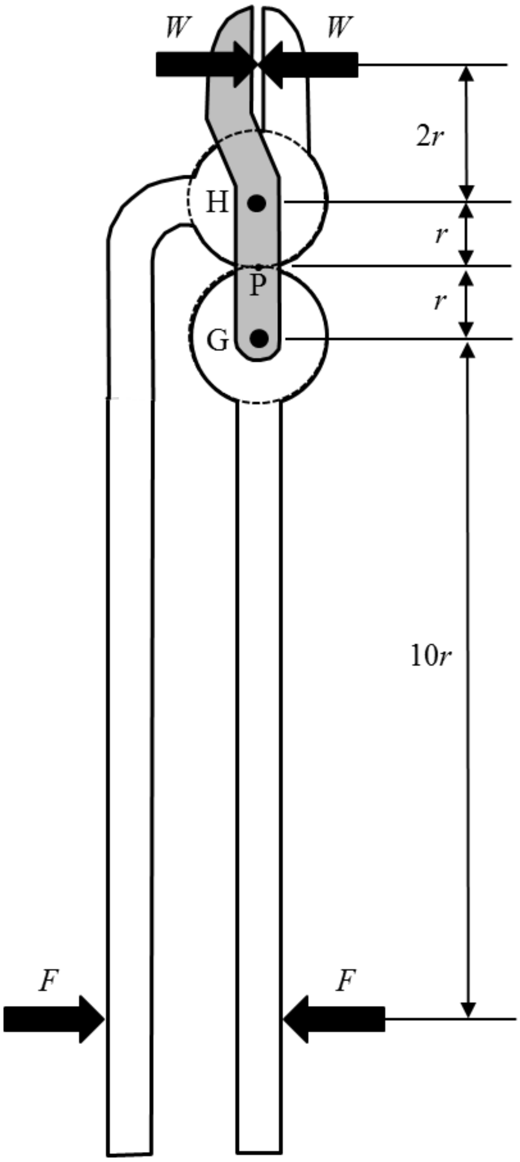

Fig. 3

(TURN OVER

3 Figure 4 shows a 'Ravigneaux' gearbox. The annulus A has 72 teeth. The two sun wheels S1 and S2 have 30 teeth and 36 teeth respectively. The carrier C supports two planet wheels P1 and P2 that mesh with each other. Each planet wheel has 18 teeth. The gearbox is fitted to a car. The road wheels are driven via the annulus. The gearbox provides four forward speed ratios and one reverse speed ratio.

(a) The first forward ratio is obtained by holding the carrier C stationary and connecting the sun S1 to the engine. Sun S2 is free to rotate. Find the speed ratio from S1 to A. [20%]

(b) The second forward ratio is obtained by holding S2 stationary and connecting S1 to the engine. Carrier C is free to rotate. Find the speed ratio from S1 to A. [20%]

(c) The third forward ratio is obtained by locking S2 to S1 (so that they rotate at the same speed) and connecting S1 or S2 to the engine. Carrier C is free to rotate. Find the speed ratio from S1 or S2 to A. [20%]

(d) The second forward ratio is selected, as in part (b). The engine speed is 2000 revolutions per minute and the input power is 50 kW. The power output from the gearbox is 48 kW. Find the torque necessary to hold S2 stationary. [30%]

(e) Explain how the reverse speed ratio could be obtained. [10%]

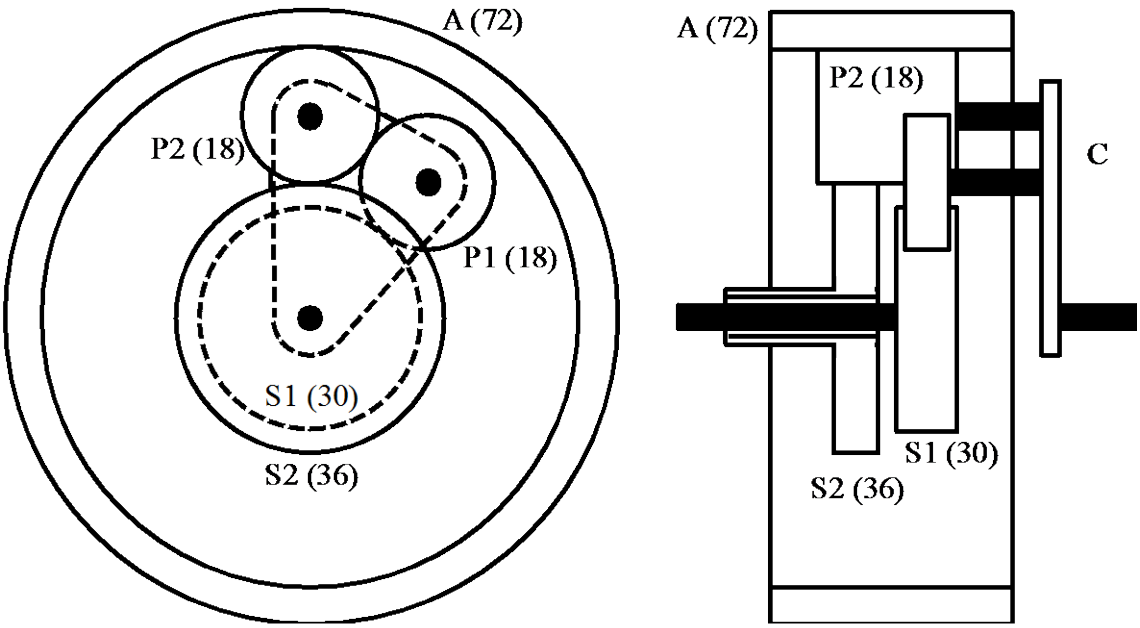

Fig. 4

4 (a) Explain carefully why Hertzian contact between two spheres can be modelled by contact between a rigid flat half-space and a sphere with an appropriate effective curvature and contact modulus. [20%]

(b) Figure 5 illustrates a sphere of radius R, plane strain Young's modulus E' and density ρ resting on a flat rigid half-space whose surface is horizontal. The contact between the bodies can be modelled by Hertz theory. The sphere rests on the half-space due to self-weight loading under the action of gravitational acceleration g. The distance between the surface of the half-space and the centre of the sphere is $R - \delta$.

(i) Derive an expression for the contact stiffness $\mathrm{d}P/\mathrm{d}\delta$ at the sphere's resting position, where P is the contact force between the sphere and the half-space, in terms of E', ρ, g and R. [20%]

(ii) Estimate the hardness of a steel sphere with radius $R = 30$ mm, $\rho = 7800$ kg m^{-3} and $E' = 230$ GPa required to avoid plastic yielding. Take g as 9.81 m s^{-2}. [25%]

(iii) The half-space is now oscillated in the vertical direction so that the height x of the surface varies with time t according to the expression

$$x = d \sin(\omega t)$$

where the amplitude d has a constant value of 2 μm. Show that the system is equivalent to case (b), section 4.6.2 in the Mechanics Data Book, and find the range of frequencies ω which will cause oscillations in δ with an amplitude of less than 0.1 μm. It may be assumed that the oscillations in δ are small compared with the mean value of δ. [35%]

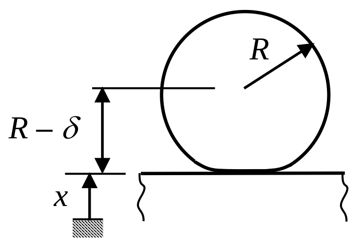

Fig. 5

END OF PAPER

THIS PAGE IS BLANK

Engineering Tripos Part IIA: Module 3C8
Machine Design

Numerical answers - 2016/17

1. Approximate answers using graphical construction. (b) 0.275, 0.825, 0.15, 63 minutes, (c) 74 minutes.

2. (a) 1100 N, (b) 1064 N, (c) 7.68 mm, 0.73 mm, (d) 0.95 GPa.

3. (a) 5/12, (b) 15/22, (c) 1, (d) 98 Nm

4 (b) (i) $2\left(\pi R^4 E'^2 \rho g\right)^{\frac{1}{3}}$, (ii) 858 MPa, (iii) 244 Hz