## Version NAF/2

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 30 April 20152 to 3.30

Module 3C9

FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than three questions.

All questions carry the same number of marks.

The approximate percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number not your name on the cover sheet.

## STATIONERY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3C9 datasheet (8 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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1 (a) Explain why the critical energy release rate for fracture, $G_{\mathrm{IC}}$, is much greater than twice the surface energy for an engineering alloy.
(b) A long, thin-walled circular tube of radius $R$ and wall thickness $t$ contains a crack of length $2 a$ through the wall of the tube as sketched in Fig. 1a. The crack is inclined at an angle $\alpha$ to the axis of the tube. Calculate the stress intensity factors $K_{I}$ and $K_{I I}$ for loading by an axial force $F$.
(c) A semi-infinite crack in a thin sheet is subjected to a concentrated load $P$ at a distance $L$ from the crack tip as shown in Fig. 1b. The tensile yield strength of the material is $\sigma_{\mathrm{Y}}$. Assuming plane stress conditions, determine:
(i) the plastic zone size using the Irwin model;
(ii) the plastic zone size according to the Dugdale model.

(a)


Fig. 1

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2 (a) Explain why the embedding of glass fibres into an aluminium sheet enhances its toughness.
(b) Aircraft wings are commonly made by adhesive bonding together a stack of thin layers of aluminium alloy. Explain why its damage tolerance exceeds that of a single sheet of the same overall thickness.
(c) A large thin sheet contains a central crack of length $2 a_{0}$, and is loaded by a uniform, in-plane tensile stress $\sigma$ normal to the crack plane. The R-curve for the sheet is

$$
K_{R}=K_{0}\left[1-\frac{1}{2} \exp (-\Delta a / \lambda)\right]
$$

where $K_{R}$ is the crack growth resistance, $\Delta a$ is the crack extension, and $K_{0}$ and $\lambda$ are material constants.
(i) Write down the value of the initiation fracture toughness, and the steady state fracture toughness.
(ii) Calculate the failure strength of the sheet if $a_{0}=\lambda$ and the associated value of crack extension $\Delta a / \lambda$.
[Hint: In order to find the root of the function $f(x)=a+x-\exp x$, note that the estimate $x_{n}$ gives rise to an improved estimate $x_{n+1}$ where $x_{n+1}=\ln \left(a+x_{n}\right)$.]

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3 Account for the following observations:
(a) Sharp notches reduce the fatigue strength of an engineering component provided the notch is sufficiently large.
(b) The slope of the S-N curve for a pre-cracked component scales with the slope of the Paris law for crack growth rate $d a / d N$ versus $\Delta K$. Also, the fatigue limit of the pre-cracked component is linked directly to the fatigue threshold.
(c) Application of an overload to a growing fatigue crack leads to major inaccuracy in integration of the Paris law.
(d) Metallic components are more flaw-sensitive to cyclic loading than to monotonic loading.
(e) The S-N curve for a smooth steel specimen is lowered in the presence of a corrosive environment or an increased tensile mean stress.

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4 (a) Explain why it is necessary to use the J-integral as a measure of the toughness for small specimens made from a material of high toughness.
(b) A cantilever beam contains a crack of length $a$ on its mid-plane, as shown in Fig. 2. Each arm of the cracked portion is subjected to a transverse end load $F$. The bending moment $M$ on either arm is related to the curvature $\kappa$ by $M=A \kappa^{N}$ where $A$ and $N$ are constants (dependent upon the beam height and non-linear plastic properties of the beam material).
(i) Obtain expressions for the distribution of bending moment $M(x)$, beam curvature $\kappa(x)$ and transverse displacement $u(x)$ of each arm as a function of distance $x$ from the loaded end of the beam.
(ii) Determine the stored energy in the arms of the beam in terms of the end load $F$, and the potential energy of the beam assuming that the end load is held fixed.
(iii) Upon recalling that the J-integral is defined as the release in potential energy per unit crack advance, obtain an expression for the critical load $F_{\mathrm{c}}$ for crack advance in terms of the toughness $J_{\mathrm{IC}}$.


Fig. 2

## END OF PAPER

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