Version NAF/4

EGT2 ENGINEERING TRIPOS PART IIA

Friday 5 May 2017 2 to 3.30

### Module 3C9

# FRACTURE MECHANICS OF MATERIALS AND STRUCTURES

Answer not more than three questions.

All questions carry the same number of marks.

The *approximate* percentage of marks allocated to each part of a question is indicated in the right margin.

Write your candidate number <u>not</u> your name on the cover sheet.

### STATIONERY REQUIREMENTS

Single-sided script paper

# SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed Attachment: 3C9 datasheet (8 pages). Engineering Data Book

# 10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain why  $K = K_{IC}$  is a valid criterion for the initiation of crack growth when small scale yielding applies. Why does the resistance to crack growth increase with crack advance for a steel? [40%]

(b) A fracture test specimen, of uniform thickness *B*, is of triangular shape, and contains a crack of length *a* along its mid-plane, as shown in Fig. 1. The half-height *h* varies with distance from the end *x* such that  $h = \alpha x$  over its length  $\ell$ . The tapered ends of the specimen are each subjected to a transverse load *P* and moment *M*.

(i) Assuming that the specimen is made from an isotropic, linear elastic solid of Young's modulus E and Poisson's ratio  $\nu$  obtain an expression for the energy release rate G in terms of the load and geometry. [40%]

(ii) If instead, only the upper arm of the specimen is loaded by a transverse load *P* and moment *M*, what is the change in *G* compared to part (i)? [20%]

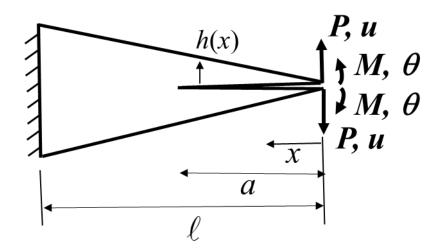


Fig. 1

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2 (a) A spherical void of initial radius  $R_0$  in an elastic, ideally plastic solid of yield strength  $\sigma_Y$  expands to a radius *R* when the solid suffers a remote uniaxial strain  $\varepsilon^{\infty}$  and a superimposed hydrostatic stress  $\sigma_h$  such that

$$\frac{R}{R_0} = \varepsilon^{\infty} \exp\left(\frac{3\sigma_h}{2\sigma_{\rm Y}}\right)$$

(i) Explain why the plane strain mode I fracture toughness for a thick plate of aluminium alloy exceeds the plane stress value for a thin sheet. [20%]

(ii) Metal Matrix Composites (MMCs) have a high volume fraction of ceramic reinforcement particles in a metallic matrix. Explain why the ductility of MMCs is sensitive to inclusion content.

(b) A frictionless, flat-bottomed punch comprises a circular cylinder of diameter D and indents a half-space as shown in Fig. 2. The solid exhibits a uniaxial nominal stress  $\sigma$  versus nominal strain  $\varepsilon$  response that obeys the power law  $\sigma = A\varepsilon^N$  in terms of a pre-exponent A and a strain hardening exponent N. A circular cylindrical crack of length a exists below the punch, as shown. Assume that an indent displacement u gives rise to a state of uniaxial compression in the cylindrical plug of height a and diameter D beneath the punch.

- (i) Calculate the punch force *P* as a function of punch displacement *u*. [20%]
- (ii) Thereby calculate the work done by the punch as a function of *u*. [20%]

(iii) Obtain an expression for the *J*-integral at the crack tip, and thereby determine the critical value of punch load for which crack advance initiates in terms of the toughness  $J_{IC}$ . [20%]

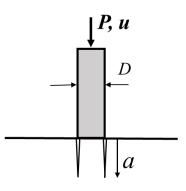


Fig. 2

[20%]

3 Account for the following observations:

(a) When a ceramic brick is suddenly immersed in liquid nitrogen from room temperature, it cracks on its faces but not in its core. In contrast, when the ceramic brick is suddenly taken from room temperature to a furnace at 1000°C it cracks within its core but not on its faces. [25%]

(b) The threshold stress intensity factor range for fatigue crack growth is sensitive to the load ratio R. [25%]

(c) Compressive-compressive loading can generate fatigue cracks in a weldment that has not been stress-relieved. [25%]

(d) The fatigue life of a railway wheel can be extended by increasing the wear rate of the wheel. [25%]

4 (a) Explain the use of Neuber's rule in predicting the fatigue life of a notched structure. What are the limitations of this approach? [30%]

(b) A steel tube of radius R and wall thickness t contains a small hole of diameter  $d \ll R$  within the wall. The tube is subjected to an axial torque Q such that the response is elastic.

(i) Show that the longitudinal shear stress  $\tau$  remote from the hole is given by  $\tau = Q/(2\pi Rt)$ . [20%]

(ii) Show that the ratio of peak tensile stress at the periphery of the hole to the shear stress  $\tau$  equals 4. You may make use of the fact that the stress concentration factor for a circular hole in a plate under remote tension equals 3 at the equator of the hole, and -1 at the poles. [20%]

(iii) The tube is now loaded cyclically from Q = 0 to  $Q = Q_{\text{max}}$ . Obtain an expression for the fatigue life using the formulae given in the Engineering Data Book, as appropriate. [30%]

# **END OF PAPER**