

EGT2

ENGINEERING TRIPOS PART IIA

Wednesday 20 April 2016 2 to 3.30

Module 3D2

GEOTECHNICAL ENGINEERING II

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS

Single-sided script paper

Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 3D1 & 3D2 Geotechnical Engineering Data Book (19 pages).

Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) When a cylindrical pressuremeter is expanded in a vertical borehole in an elastic-perfectly plastic clay under undrained conditions, the clay around the pressuremeter behaves elastically until the cavity pressure, σ_c , reaches a certain value, above which a plastic zone develops in the clay around the pressuremeter.

(i) Show that the shear stress in the clay immediately adjacent to the pressuremeter membrane reaches its yield strength when the volumetric expansion is given by

$$\frac{\Delta V}{V} = \frac{c_u}{G}$$

where c_u is the undrained shear strength and G is the elastic shear modulus of the clay. [20%]

(ii) By assuming that the effective stresses remain constant in the plastic zone so that the increase in cavity pressure is matched by an increase in pore pressure at the wall of the cavity, use the Data Book to show that the excess pore pressure, Δu , generated at the cavity wall is given by

$$\Delta u = c_u \ln \left[\frac{G}{c_u} \frac{\Delta V}{V} \right] \quad [20\%]$$

(b) A self-boring pressuremeter test was carried out in soft clay at a depth of 12 m. The water table was 1.5 m below the ground surface and the unit weight of the soil was 15 kN m^{-3} throughout its depth. The test results are given below in terms of the cavity pressure, σ_c , and the cavity strain, ε_c , which is defined as the radial displacement of the pressuremeter divided by the initial radius.

Cavity pressure (kN m^{-2})	Cavity strain (%)
0	0
75	0
150	0
160	0
180	0.1
190	0.2
205	0.5
230	1.2
255	2.5
270	4.1
290	7.7
300	10.5

- (i) Estimate the coefficient of horizontal earth pressure at rest, K_0 , defined in terms of effective stresses. [20%]
- (ii) By plotting σ_c versus $\ln \varepsilon_c$ estimate the undrained shear strength of the clay. [20%]
- (iii) Assuming the elastic shear modulus, G , of the clay to be 4500 kN m^{-2} , estimate the excess pore pressure that would have been measured at the cavity wall when the pressuremeter had been expanded to a cavity strain $\varepsilon_c = 10\%$. [20%]

2 Figure 1 shows a site on which two future projects are to be designed: (a) a retaining wall for a metro station, and (b) a warehouse building on shallow foundations. Plane strain conditions are to be assumed. The ground is firm overconsolidated sandy clay, for which the unit weight $\gamma = 17.5 \text{ kN m}^{-3}$ (above and below the water table) and the angle of friction $\phi_{crit} = 28^\circ$. The ground water table is 3 m below ground level. At the location of each project several high quality samples of clay have been taken from a depth of 10 m. Self-boring pressuremeter tests have established that at this depth the coefficient of earth pressure at rest in terms of effective stresses, $K_0 = 0.9$.

(a) Calculate the effective and total vertical and horizontal in-situ stresses at a depth of 10 m and express these in terms of t , s' and s , as defined in the Data Book. Using the Data Book, what was the original value of $K_0 = K_{onc}$ when the clay was deposited under normally consolidated conditions? [20%]

(b) Two plane strain biaxial tests were undertaken on the samples. In each case the present day effective and total in-situ stresses were imposed on the samples before they were subjected to additional shearing. Plot the effective and total stress paths in terms of t , s' and s (as defined in the Data Book) for each test, details of which are as follows, and answer the following questions:

(i) Test A: an undrained test to simulate rapid construction of the warehouse; the vertical total stress was increased, while maintaining the horizontal total stress constant, until the sample reached failure. The resulting undrained shear strength, c_u , was found to be 60 kN m^{-2} . What was the measured pore pressure at failure? Comment on the effective stress path in the earlier stage of the test. [40%]

(ii) Test B: an undrained test to simulate rapid excavation in front of the retaining wall; the horizontal total stress was reduced, while maintaining the vertical total stress constant, until the sample reached failure. The measured undrained shear strength was the same as for Test A. What was the measured pore pressure at failure? [40%]

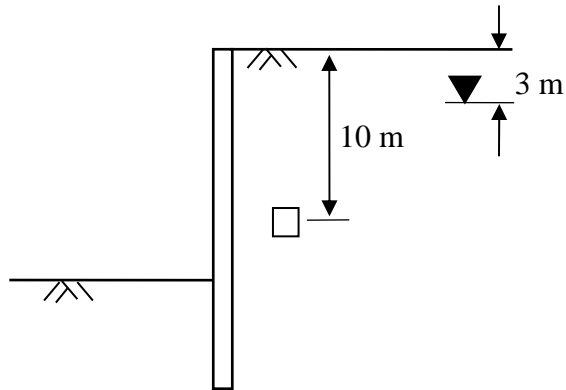


Fig.1(a)

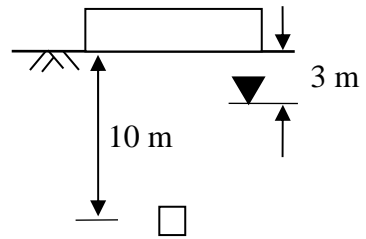


Fig.1(b)

3 An infinite slope of sand with an angle of 30 degrees is shown in Fig. 2(a). The soil is a carbonate sand with a critical state friction angle of 39°. The maximum and minimum void ratios for the sand are 0.95 and 0.6 respectively. The uppermost 5 m of the soil is well compacted with a void ratio of 0.7, whereas beneath this layer the soil is less well compacted having a void ratio of 0.8. The specific gravity of the sand is 2.75.

- (a) Calculate the dry and saturated unit weights of the surface soil layer. [10%]
- (b) Sketch the shear stress-displacement curves for loose and dense sands as observed in a direct shear test. Describe the micromechanics behind the shapes of the curves and discuss the use of the strength parameters in design. [20%]
- (c) If the slope is dry, find the stress state of a soil element at the base of the well compacted layer. Plot the stress state on a (τ, σ') diagram and discuss the stability of the slope by considering the strength of the soil. [25%]
- (d) If the same slope is subjected to heavy rainfall and the water table rises, remaining parallel to the ground surface at a depth d below the surface as shown in Fig. 2(b). Find the total and effective stresses at the base of the well compacted layer as a function of d . Plot the stress path as the water table rises on a (τ, σ') diagram and discuss the stability of the slope. [45%]

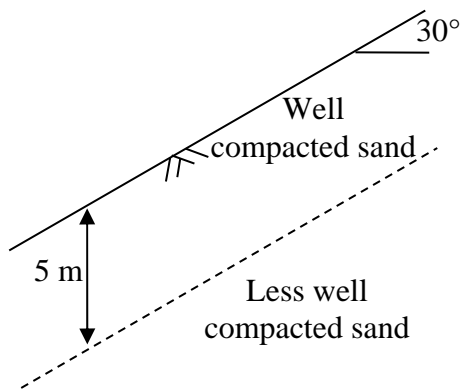


Fig. 2(a)

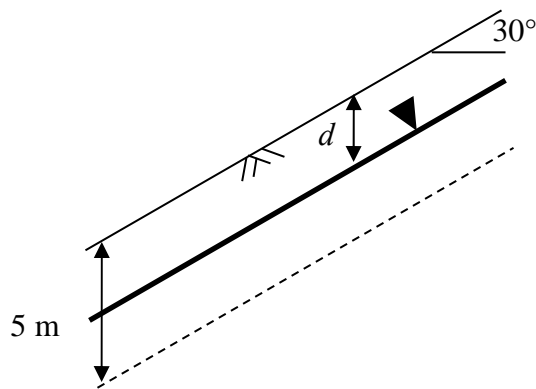


Fig. 2(b)

4 A soft clay is found to have properties similar to those listed for kaolin in the Data Book. Construction of a station box in this clay requires excavation to a characteristic depth of 20 m, in a location where the water table lies 5 m below the ground surface.

(a) What are the main challenges of working with clays whose specific volume lies above the Critical State Line on a $(v, \ln(\sigma'))$ diagram? Illustrate your answer with stress paths and offer a micromechanical explanation for the behaviour. [20%]

(b) Oedometer tests on samples recovered from 20 m depth suggest that the soil is lightly overconsolidated with an effective preconsolidation stress of 250 kPa. Sketch possible stress histories on a $(v, \ln(\sigma'))$ diagram for soil elements at 10 m and 20 m depth. Estimate the current specific volumes and unit weights of the soil at these depths. [25%]

(c) A soil element from 20 m depth is subjected to a drained compression triaxial test under a cell pressure of 150 kPa. Sketch this effective stress path on (q, p') and $(v, \ln(p'))$ diagrams. Calculate the yield stress and ultimate drained strength of the soil at 20 m depth using Cam Clay and mark corresponding points Y_D and F_D on both diagrams. What is the volumetric strain at failure? [35%]

(d) If the soil element at 20 m depth is instead subjected to undrained triaxial loading with a cell pressure of 150 kPa, sketch the effective stress path on both diagrams and calculate the yield stress and ultimate undrained strength of the soil using Cam Clay and mark corresponding points Y_U and F_U on both diagrams. [20%]

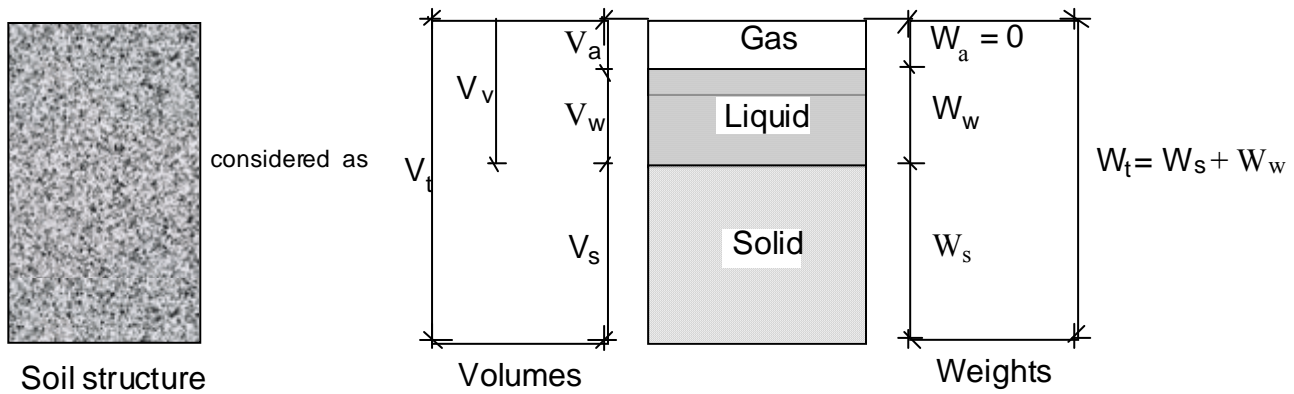
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Engineering Tripos Part IIA**3D1 & 3D2
Geotechnical Engineering
Data Book 2012-2013**

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General definitions



Specific gravity of solid

$$G_s$$

Voids ratio

$$e = V_v / V_s$$

Specific volume

$$v = V_t / V_s = 1 + e$$

Porosity

$$n = V_v / V_t = e / (1 + e)$$

Water content

$$w = (W_w / W_s)$$

Degree of saturation

$$S_r = V_w / V_v = (w G_s / e)$$

Unit weight of water

$$\gamma_w = 9.81 \text{ kN/m}^3$$

Unit weight of soil

$$\gamma = W_t / V_t = \left(\frac{G_s + S_r e}{1 + e} \right) \gamma_w$$

Buoyant saturated unit weight

$$\gamma' = \gamma - \gamma_w = \left(\frac{G_s - 1}{1 + e} \right) \gamma_w$$

Unit weight of dry solids

$$\gamma_d = W_s / V_t = \left(\frac{G_s}{1 + e} \right) \gamma_w$$

Air volume ratio

$$A = V_a / V_t = \left(\frac{e(1 - S_r)}{1 + e} \right)$$

Soil classification (BS1377)Liquid limit w_L Plastic Limit w_P Plasticity Index $I_P = w_L - w_P$ Liquidity Index $I_L = \frac{w - w_P}{w_L - w_P}$ Activity = $\frac{\text{Plasticity Index}}{\text{Percentage of particles finer than } 2 \mu\text{m}}$ Sensitivity = $\frac{\text{Unconfined compressive strength of an undisturbed specimen}}{\text{Unconfined compressive strength of a remoulded specimen}}$ (at the same water content)*Classification of particle sizes:-*

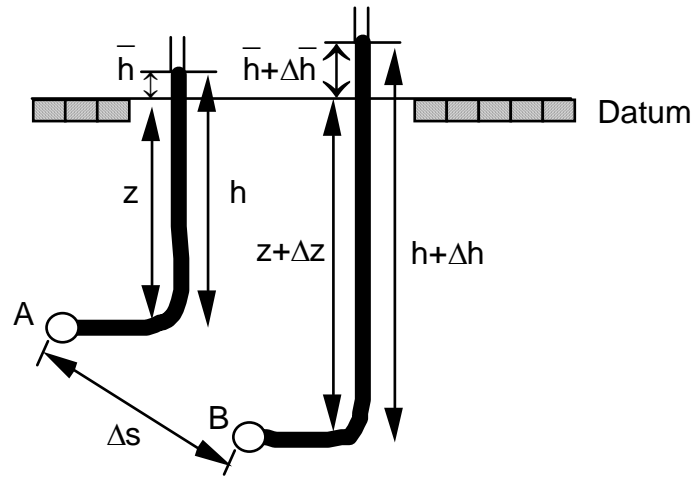
Boulders	larger than			200 mm
Cobbles	between	200 mm	and	60 mm
Gravel	between	60 mm	and	2 mm
Sand	between	2 mm	and	0.06 mm
Silt	between	0.06 mm	and	0.002 mm
Clay	smaller than	0.002 mm (two microns)		

D equivalent diameter of soil particle

 D_{10} , D_{60} etc. particle size such that 10% (or 60%) etc.) by weight of a soil sample is composed of finer grains. C_U uniformity coefficient D_{60}/D_{10}

Seepage

Flow potential:
(piezometric level)



Total gauge pore water pressure at A: $u = \gamma_w h = \gamma_w (\bar{h} + z)$

$$\text{B: } u + \Delta u = \gamma_w (h + \Delta h) = \gamma_w (\bar{h} + z + \Delta \bar{h} + \Delta z)$$

Excess pore water pressure at A: $\bar{u} = \gamma_w \bar{h}$

$$\text{B: } \bar{u} + \Delta \bar{u} = \gamma_w (\bar{h} + \Delta \bar{h})$$

Hydraulic gradient A \rightarrow B $i = -\frac{\Delta \bar{h}}{\Delta s}$

Hydraulic gradient (3D) $i = -\nabla \bar{h}$

Darcy's law $V = ki$

V = superficial seepage velocity

k = coefficient of permeability

Typical permeabilities:

$D_{10} > 10 \text{ mm}$:	non-laminar flow
$10 \text{ mm} > D_{10} > 1 \mu\text{m}$:	$k \cong 0.01 (D_{10} \text{ in mm})^2 \text{ m/s}$
clays	:	$k \cong 10^{-9} \text{ to } 10^{-11} \text{ m/s}$

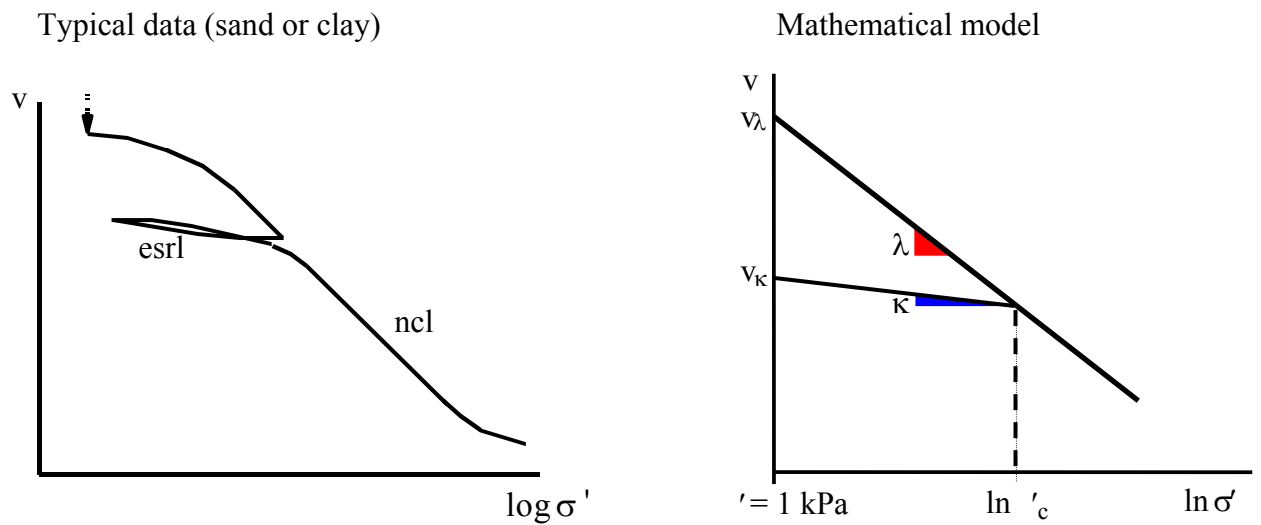
Saturated capillary zone

$$h_c = \frac{4T}{\gamma_w d} \quad : \quad \text{capillary rise in tube diameter } d, \text{ for surface tension } T$$

$$h_c \approx \frac{3 \times 10^{-5}}{D_{10}} \text{ m} \quad : \quad \text{for water at } 10^\circ\text{C}; \text{ note air entry suction is } u_c = -\gamma_w h_c$$

One-Dimensional Compression

• Fitting data



Plastic compression stress σ'_c is taken as the larger of the initial aggregate crushing stress and the historic maximum effective vertical stress. Clay muds are taken to begin with $\sigma'_c \approx 1$ kPa.

Plastic compression (normal compression line, ncl): $v = v_\lambda - \lambda \ln \sigma'$ for $\sigma' = \sigma'_c$

Elastic swelling and recompression line (esrl):
 $v = v_c + \kappa (\ln \sigma'_c - \ln \sigma'_v)$
 $= v_\kappa - \kappa \ln \sigma'_v$ for $\sigma' < \sigma'_c$

Equivalent parameters for \log_{10} stress scale:

Terzaghi's compression index $C_c = \lambda \log_{10} e$

Terzaghi's swelling index $C_s = \kappa \log_{10} e$

• Deriving confined soil stiffnesses

Secant 1D compression modulus $E_o = (\Delta \sigma' / \Delta \epsilon)_o$

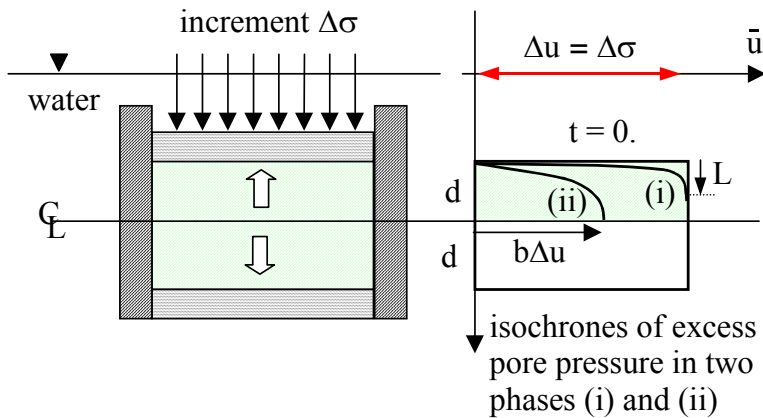
Tangent 1D plastic compression modulus $E_o = v \sigma' / \lambda$

Tangent 1D elastic compression modulus $E_o = v \sigma' / \kappa$

One-Dimensional Consolidation

Settlement	ρ	$= \int m_v (\Delta u - \bar{u}) dz$	$= \int (\Delta u - \bar{u}) / E_o dz$
Coefficient of consolidation	c_v	$= \frac{k}{m_v \gamma_w}$	$= \frac{kE_o}{\gamma_w}$
Dimensionless time factor	T_v	$= \frac{c_v t}{d^2}$	
Relative settlement	R_v	$= \frac{\rho}{\rho_{ult}}$	

• Solutions for initially rectangular distribution of excess pore pressure



Approximate solution by parabolic isochrones:

Phase (i) $L^2 = 12 c_v t$
 $R_v = \sqrt{\frac{4T_v}{3}}$ for $T_v < 1/12$

Phase (ii) $b = \exp(1/4 - 3T_v)$
 $R_v = [1 - 2/3 \exp(1/4 - 3T_v)]$ for $T_v > 1/12$

Solution by Fourier Series:

T_v	0	0.01	0.02	0.04	0.08	0.15	0.20	0.30	0.40	0.50	0.60	0.80	1.00
R_v	0	0.12	0.17	0.23	0.32	0.45	0.51	0.62	0.70	0.77	0.82	0.89	0.94

Stress and strain components

- **Principle of effective stress (saturated soil)**

$$\text{total stress } \sigma = \text{effective stress } \sigma' + \text{pore water pressure } u$$

- **Principal components of stress and strain**

sign convention	compression positive
total stress	$\sigma_1, \sigma_2, \sigma_3$
effective stress	$\sigma'_1, \sigma'_2, \sigma'_3$
strain	$\varepsilon_1, \varepsilon_2, \varepsilon_3$

- **Simple Shear Apparatus (SSA)** ($\varepsilon_2 = 0$; other principal directions unknown)

The only stresses that are readily available are the shear stress τ and normal stress σ applied to the top platen. The pore pressure u can be controlled and measured, so the normal effective stress σ' can be found. Drainage can be permitted or prevented. The shear strain γ and normal strain ε are measured with respect to the top platen, which is a plane of zero extension. Zero extension planes are often identified with slip surfaces.

$$\text{work increment per unit volume} \quad \delta W = \tau \delta\gamma + \sigma' \delta\varepsilon$$

- **Biaxial Apparatus - Plane Strain (BA-PS)** ($\varepsilon_2 = 0$; rectangular edges along principal axes)

Intermediate principal effective stress σ'_2 , in zero strain direction, is frequently unknown so that all conditions are related to components in the 1-3 plane.

mean total stress	$s = (\sigma_1 + \sigma_3)/2$
mean effective stress	$s' = (\sigma'_1 + \sigma'_3)/2 = s - u$
shear stress	$t = (\sigma'_1 - \sigma'_3)/2 = (\sigma_1 - \sigma_3)/2$

volumetric strain	$\varepsilon_v = \varepsilon_1 + \varepsilon_3$
shear strain	$\varepsilon_\gamma = \varepsilon_1 - \varepsilon_3$

$$\text{work increment per unit volume} \quad \delta W = \sigma'_1 \delta\varepsilon_1 + \sigma'_3 \delta\varepsilon_3$$

$$\delta W = s' \delta\varepsilon_v + t \delta\varepsilon_\gamma$$

providing that principal axes of strain increment and of stress coincide.

• **Triaxial Apparatus – Axial Symmetry (TA-AS)** (cylindrical element with radial symmetry)

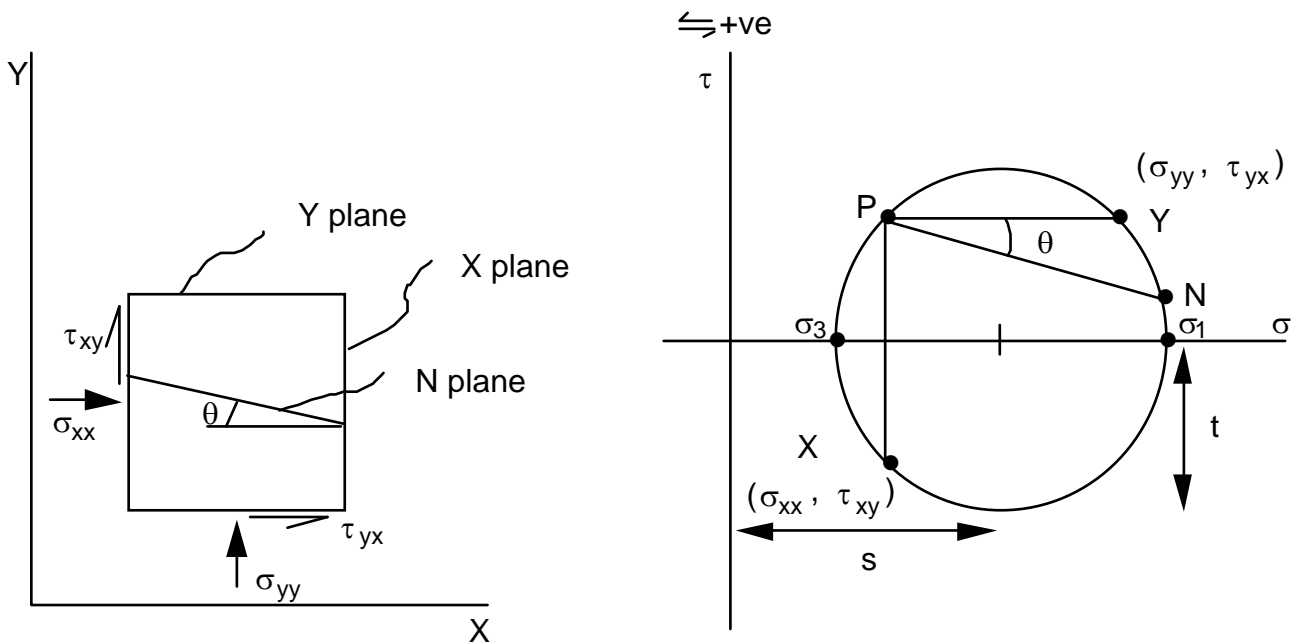
total axial stress	$\sigma_a = \sigma'_a + u$
total radial stress	$\sigma_r = \sigma'_r + u$
total mean normal stress	$p = (\sigma_a + 2\sigma_r)/3$
effective mean normal stress	$p' = (\sigma'_a + 2\sigma'_r)/3 = p - u$
deviatoric stress	$q = \sigma'_a - \sigma'_r = \sigma_a - \sigma_r$
stress ratio	$\eta = q/p'$
axial strain	ϵ_a
radial strain	ϵ_r
volumetric strain	$\epsilon_v = \epsilon_a + 2\epsilon_r$
triaxial shear strain	$\epsilon_s = \frac{2}{3}(\epsilon_a - \epsilon_r)$
work increment per unit volume	$\delta W = \sigma'_a \delta \epsilon_a + 2\sigma'_r \delta \epsilon_r$
	$\delta W = p' \delta \epsilon_v + q \delta \epsilon_s$

Types of triaxial test include:

- isotropic compression* in which p' increases at zero q
- triaxial compression* in which q increases *either* by increasing σ_a *or* by reducing σ_r
- triaxial extension* in which q reduces *either* by reducing σ_a *or* by increasing σ_r

• **Mohr's circle of stress (1–3 plane)**

Sign of convention: compression, and counter-clockwise shear, positive



Poles of planes P : the components of stress on the N plane are given by the intersection N of the Mohr circle with the line PN through P parallel to the plane.

Elastic stiffness relations

These relations apply to tangent stiffnesses of over-consolidated soil, with a state point on some swelling and recompression line (κ -line), and remote from gross plastic yielding.

One-dimensional compression (axial stress and strain increments $d\sigma'$, $d\varepsilon$)

$$\text{compressibility} \quad m_v = \frac{d\varepsilon}{d\sigma'}$$

$$\text{constrained modulus} \quad E_o = \frac{1}{m_v}$$

Physically fundamental parameters

$$\text{shear modulus} \quad G' = \frac{dt}{d\varepsilon_\gamma}$$

$$\text{bulk modulus} \quad K' = \frac{dp'}{d\varepsilon_v}$$

Parameters which can be used for constant-volume deformations

$$\text{undrained shear modulus} \quad G_u = G'$$

$$\text{undrained bulk modulus} \quad K_u = \infty \quad (\text{neglecting compressibility of water})$$

Alternative convenient parameters

$$\text{Young's moduli} \quad E' \text{ (effective), } E_u \text{ (undrained)}$$

$$\text{Poisson's ratios} \quad \nu' \text{ (effective), } \nu_u = 0.5 \text{ (undrained)}$$

Typical value of Poisson's ratio for small changes of stress: $\nu' = 0.2$

$$\text{Relationships:} \quad G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

$$E_o = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}$$

Cam Clay

- Interchangeable parameters for stress combinations at yield, and plastic strain increments

System	Effective normal stress	Plastic normal strain	Effective shear stress	Plastic shear strain	Critical stress ratio	Plastic normal stress	Critical normal stress
General	σ^*	ε^*	τ^*	γ^*	μ^*_{crit}	σ^*_c	σ^*_{crit}
SSA	σ'	ε	τ	γ	$\tan \phi_{crit}$	σ'_c	σ'_{crit}
BA-PS	s'	ε_v	t	ε_γ	$\sin \phi_{crit}$	s'_c	s'_{crit}
TA-AS	p'	ε_v	q	ε_s	M	p'_c	p'_{crit}

- General equations of plastic work

Plastic work and dissipation

$$\sigma^* \delta\varepsilon^* + \tau^* \delta\gamma^* = \mu^*_{crit} \sigma^* \delta\gamma^*$$

Plastic flow rule – normality

$$\frac{d\tau^*}{d\sigma^*} \cdot \frac{d\gamma^*}{d\varepsilon^*} = -1$$

- General yield surface

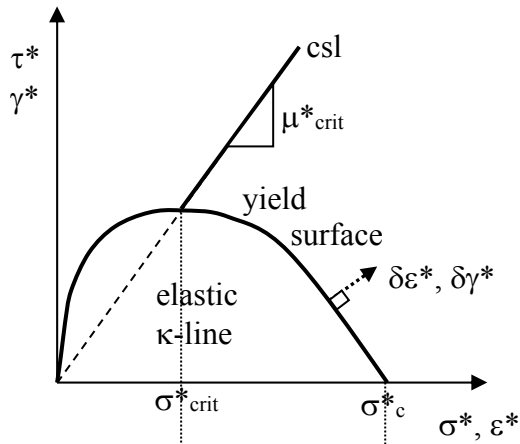
$$\frac{\tau^*}{\sigma^*} = \mu^* = \mu^*_{crit} \cdot \ln \left[\frac{\sigma^*_c}{\sigma^*} \right]$$

- Parameter values which fit soil data

	London Clay	Weald Clay	Kaolin	Dog's Bay Sand	Ham River Sand
λ^*	0.161	0.093	0.26	0.334	0.163
κ^*	0.062	0.035	0.05	0.009	0.015
Γ^* at 1 kPa	2.759	2.060	3.767	4.360	3.026
$\sigma^*_{c, virgin}$ kPa	1	1	1	Loose 500 Dense 1500	Loose 2500 Dense 15000
ϕ_{crit}	23°	24°	26°	39°	32°
M_{comp}	0.89	0.95	1.02	1.60	1.29
M_{extn}	0.69	0.72	0.76	1.04	0.90
w_L	0.78	0.43	0.74	-----	-----
w_P	0.26	0.18	0.42	-----	-----
G_s	2.75	2.75	2.61	2.75	2.65

Note: 1) parameters λ^* , κ^* , Γ^* , $\sigma^*_{c, virgin}$ should depend to a small extent on the deformation mode, e.g. SSA, BA-PS, TA-AS, etc. This may be neglected unless further information is given.
 2) Sand which is loose, or loaded cyclically, compacts more than Cam Clay allows.

• The yield surface in (σ^*, τ^*, v) space



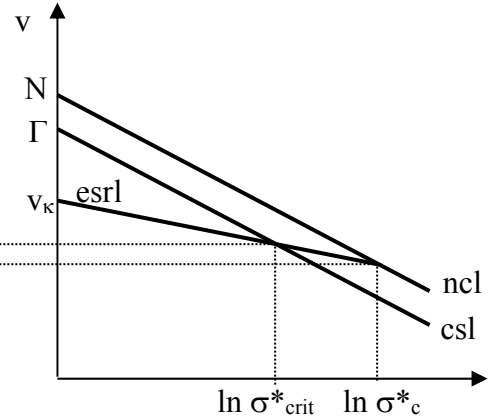
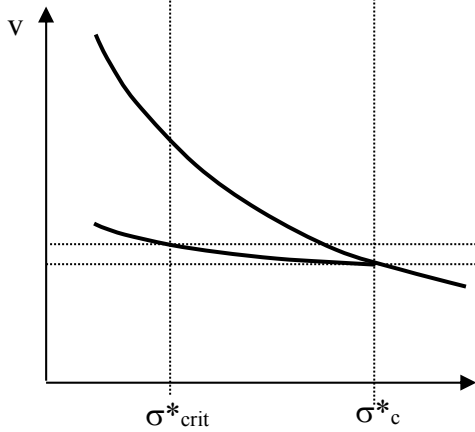
ncl: normal compression line

$$v = N - \lambda \ln \sigma^*$$

csl: critical state line

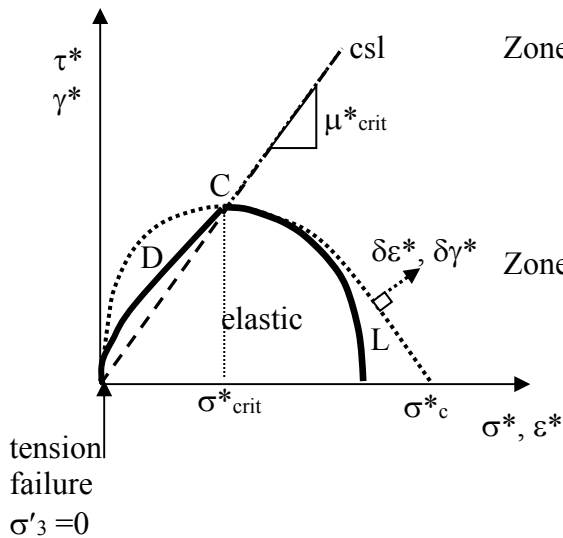
$$v = \Gamma - \lambda \ln \sigma^*$$

where $N = \Gamma + \lambda - \kappa$



• Regions of limiting soil behaviour

Variation of Cam Clay yield surface

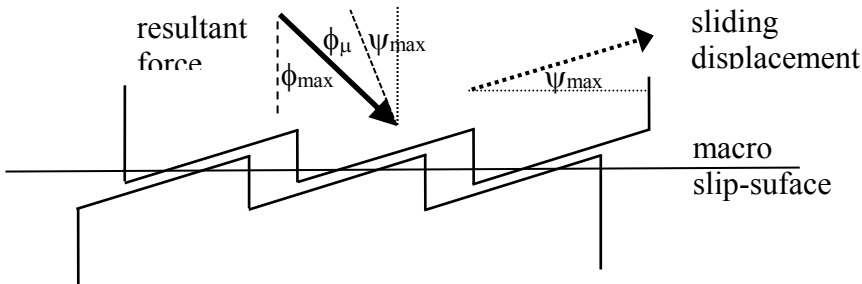


Zone D: denser than critical, “dry”,
dilation or negative excess pore pressures,
Hvorslev strength envelope,
friction-dilatancy theory,
unstable shear rupture, progressive failure

Zone L: looser than critical, “wet”,
compaction or positive excess pore pressures,
Modified Cam Clay yield surface,
stable strain-hardening continuum

Strength of soil: friction and dilation

- Friction and dilatancy: the saw-blade model of direct shear

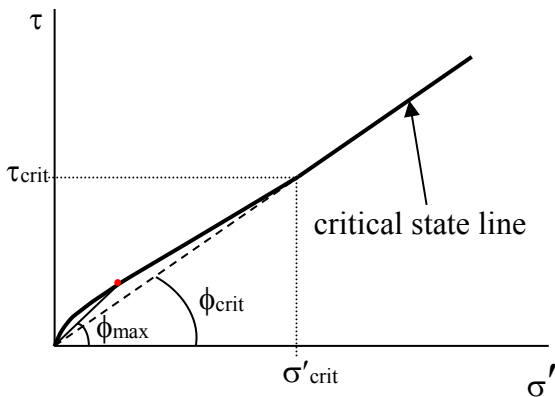


Intergranular angle of friction at sliding contacts ϕ_μ

Angle of dilation ψ_{\max}

Angle of internal friction $\phi_{\max} = \phi_\mu + \psi_{\max}$

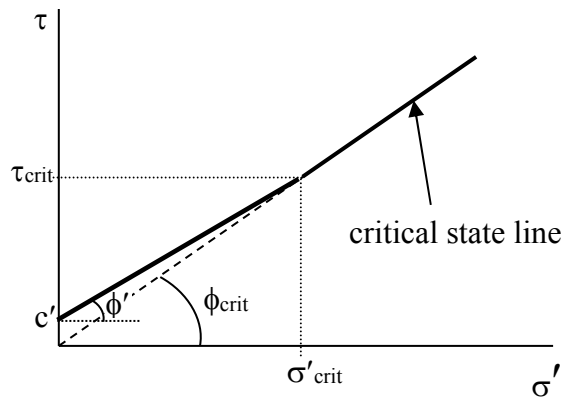
- Friction and dilatancy: secant and tangent strength parameters



Secant angle of internal friction

$$\begin{aligned} \tau &= \sigma' \tan \phi_{\max} \\ \phi_{\max} &= \phi_{\text{crit}} + \Delta\phi \\ \Delta\phi &= f(\sigma'_{\text{crit}}/\sigma') \end{aligned}$$

typical envelope fitting data:
power curve
 $(\tau/\tau_{\text{crit}}) = (\sigma'/\sigma'_{\text{crit}})^\alpha$
with $\alpha \approx 0.85$



Tangent angle of shearing envelope

$$\begin{aligned} \tau &= c' + \sigma' \tan \phi' \\ c' &= f(\sigma'_{\text{crit}}) \end{aligned}$$

typical envelope:
straight line
 $\tan \phi' = 0.85 \tan \phi_{\text{crit}}$
 $c' = 0.15 \tau_{\text{crit}}$

• **Friction and dilation: data of sands**

The inter-granular friction angle of quartz grains, $\phi_{\mu} \approx 26^{\circ}$. Turbulent shearing at a critical state causes ϕ_{crit} to exceed this. The critical state angle of internal friction ϕ_{crit} is a function of the uniformity of particle sizes, their shape, and mineralogy, and is developed at large shear strains irrespective of initial conditions. Typical values of $\phi_{\text{crit}} (\pm 2^{\circ})$ are:

well-graded, angular quartz or feldspar sands	40°
uniform sub-angular quartz sand	36°
uniform rounded quartz sand	32°

Relative density $I_D = \frac{(e_{\text{max}} - e)}{(e_{\text{max}} - e_{\text{min}})}$ where:

e_{max} is the maximum void ratio achievable in quick-tilt test

e_{min} is the minimum void ratio achievable by vibratory compaction

Relative crushability $I_C = \ln(\sigma_c / p')$ where:

σ_c is the aggregate crushing stress, taken to be a material constant, typical values being: 80 000 kPa for quartz silt, 20 000 kPa for quartz sand, 5 000 kPa for carbonate sand.

p' is the mean effective stress at failure which may be taken as approximately equal to the effective stress σ' normal to a shear plane.

Dilatancy contribution to the peak angle of internal friction is $\Delta\phi = (\phi_{\text{max}} - \phi_{\text{crit}}) = f(I_R)$

Relative dilatancy index $I_R = I_D I_C - 1$ where:

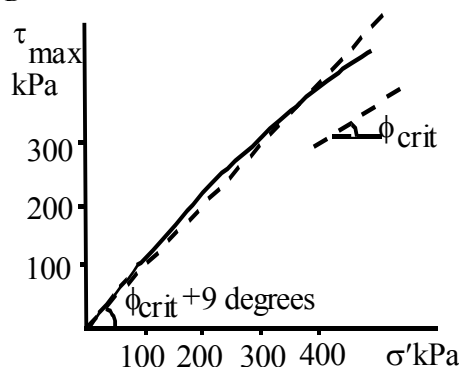
$I_R < 0$ indicates compaction, so that I_D increases and $I_R \rightarrow 0$ ultimately at a critical state

$I_R > 4$ to be limited to $I_R = 4$ unless corroborative dilatant strength data is available

The following empirical correlations are then available

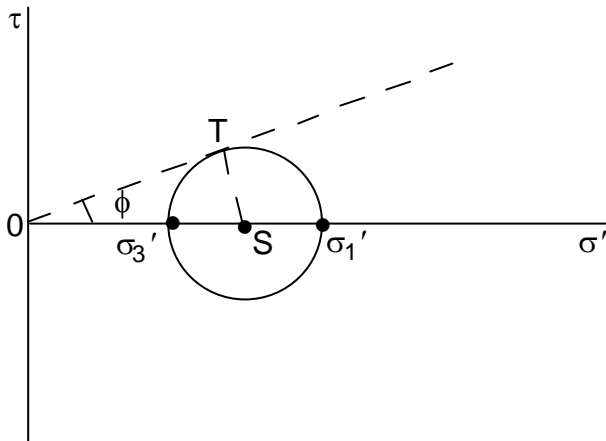
plane strain conditions	$(\phi_{\text{max}} - \phi_{\text{crit}})$	=	$0.8 \psi_{\text{max}}$	=	$5 I_R$ degrees
triaxial strain conditions	$(\phi_{\text{max}} - \phi_{\text{crit}})$	=	$3 I_R$ degrees		
all conditions	$(-\delta\varepsilon_v / \delta\varepsilon_1)_{\text{max}}$	=	$0.3 I_R$		

The resulting peak strength envelope for triaxial tests on a quartz sand at an initial relative density $I_D = 1$ is shown below for the limited stress range 10 - 400 kPa:



$$\phi_{\text{max}} > \phi_{\text{crit}} + 9^{\circ} \quad \text{for } I_D = 1, \sigma' = 400 \text{ kPa}$$

• Mobilised (secant) angle of shearing ϕ in the 1 – 3 plane



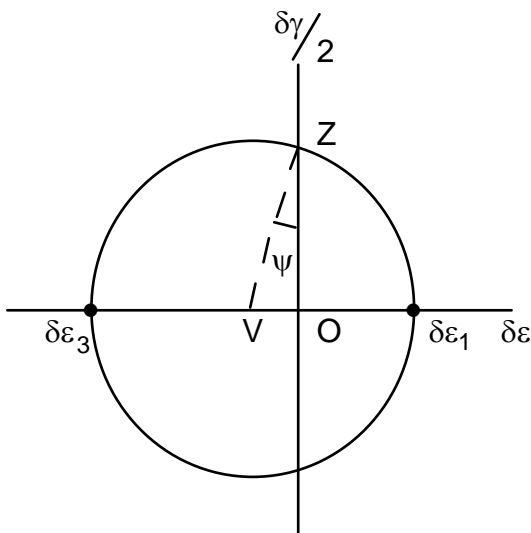
$$\begin{aligned} \sin \phi &= TS/OS \\ &= \frac{(\sigma'_1 - \sigma'_3)/2}{(\sigma'_1 + \sigma'_3)/2} \\ \left[\frac{\sigma'_1}{\sigma'_3} \right] &= \frac{(1 + \sin \phi)}{(1 - \sin \phi)} \end{aligned}$$

Angle of shearing resistance:

at peak strength ϕ_{\max} at $\left[\frac{\sigma'_1}{\sigma'_3} \right]_{\max}$

at critical state ϕ_{crit} after large shear strains

• Mobilised angle of dilation in plane strain ψ in the 1 – 3 plane



$$\begin{aligned} \sin \psi &= VO/VZ \\ &= -\frac{(\delta \epsilon_1 + \delta \epsilon_3)/2}{(\delta \epsilon_1 - \delta \epsilon_3)/2} \\ &= -\frac{\delta \epsilon_v}{\delta \epsilon_\gamma} \\ \left[\frac{\delta \epsilon_1}{\delta \epsilon_3} \right] &= -\frac{(1 - \sin \psi)}{(1 + \sin \psi)} \end{aligned}$$

at peak strength $\psi = \psi_{\max}$ at $\left[\frac{\sigma'_1}{\sigma'_3} \right]_{\max}$

at critical state $\psi = 0$ since volume is constant

Plasticity: Cohesive material $\tau_{max} = c_u$ (or s_u)

• **Limiting stresses**

Tresca $|\sigma_1 - \sigma_3| = q_u = 2c_u$

von Mises $(\sigma_1 - p)^2 + (\sigma_2 - p)^2 + (\sigma_3 - p)^2 = \frac{2}{3} q_u^2 = 2c_u^2$

where q_u is the undrained triaxial compression strength, and c_u is the undrained plane shear strength.

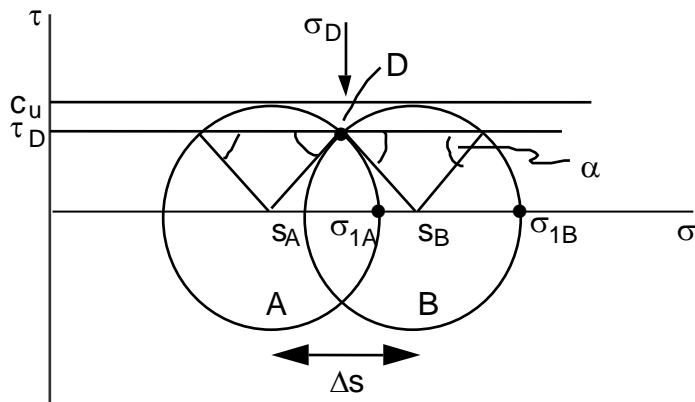
Dissipation per unit volume in plane strain deformation following either Tresca or von Mises,

$$\delta D = c_u \delta \epsilon_\gamma$$

For a relative displacement x across a slip surface of area A mobilising shear strength c_u , this becomes

$$D = Ac_u x$$

• **Stress conditions across a discontinuity**



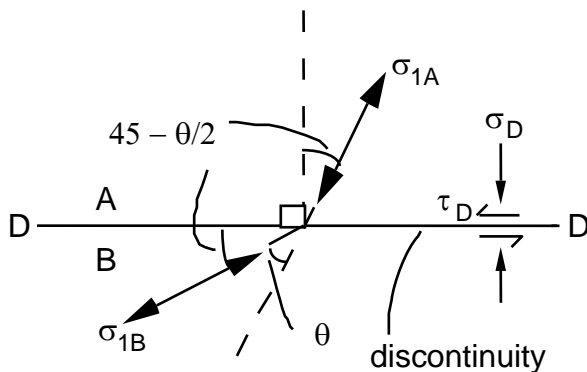
Rotation of major principal stress θ

$$s_B - s_A = \Delta s = 2c_u \sin \theta$$

$$\sigma_{1B} - \sigma_{1A} = 2c_u \sin \theta$$

In limit with $\theta \rightarrow 0$

$$ds = 2c_u d\theta$$



Useful example:

$$\theta = 30^\circ$$

$$\sigma_{1B} - \sigma_{1A} = c_u$$

$$\tau_D / c_u = 0.87$$

σ_{1A} = major principal stress in zone A

σ_{1B} = major principal stress in zone B

Plasticity: Frictional material $(\tau/\sigma')_{\max} = \tan \phi$

• **Limiting stresses**

$$\sin \phi = (\sigma'_{1f} - \sigma'_{3f}) / (\sigma'_{1f} + \sigma'_{3f}) = (\sigma_{1f} - \sigma_{3f}) / (\sigma_{1f} + \sigma_{3f} - 2u_s)$$

where σ'_{1f} and σ'_{3f} are the major and minor principal effective stresses at failure, σ_{1f} and σ_{3f} are the major and minor principle total stresses at failure, and u_s is the steady state pore pressure.

Active pressure:

$$\sigma'_v > \sigma'_h$$

$$\sigma'_1 = \sigma'_v \text{ (assuming principal stresses are horizontal and vertical)}$$

$$\sigma'_3 = \sigma'_h$$

$$K_a = (1 - \sin \phi) / (1 + \sin \phi)$$

Passive pressure:

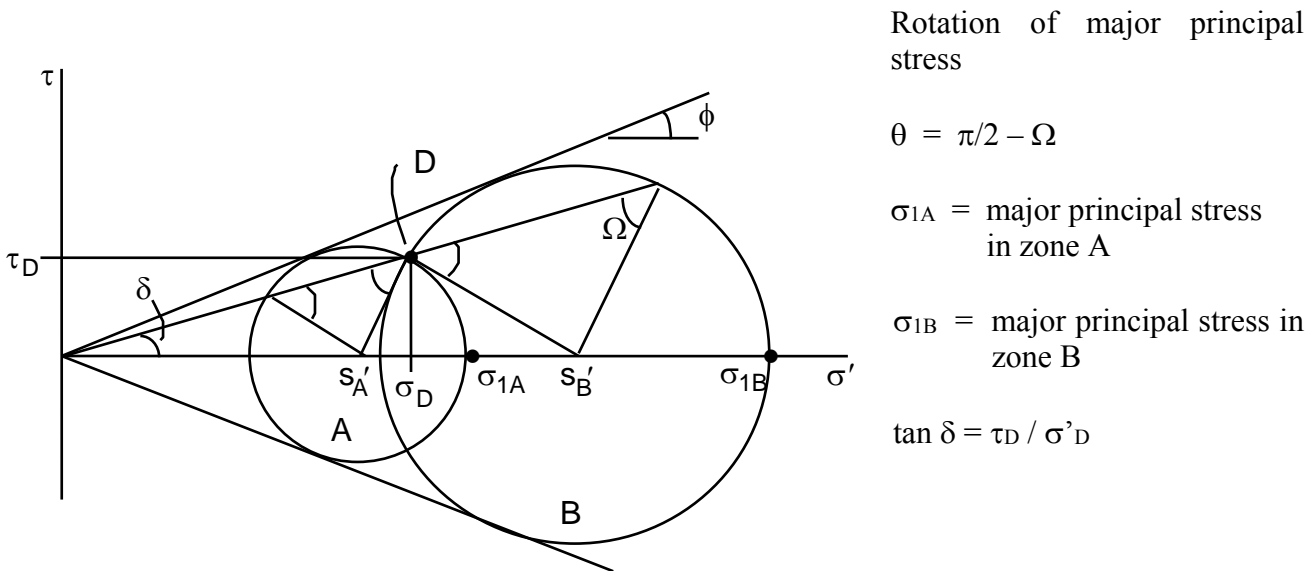
$$\sigma'_h > \sigma'_v$$

$$\sigma'_1 = \sigma'_h \text{ (assuming principal stresses are horizontal and vertical)}$$

$$\sigma'_3 = \sigma'_v$$

$$K_p = (1 + \sin \phi) / (1 - \sin \phi) = 1 / K_a$$

• **Stress conditions across a discontinuity**



$$\sin \Omega = \sin \delta / \sin \phi$$

$$s'_B / s'_A = \sin(\Omega + \delta) / \sin(\Omega - \delta)$$

In limit, $d\theta \rightarrow 0$ and $\delta \rightarrow \phi$

$$ds' = 2s' \cdot d\theta \tan \phi$$

Integration gives $s'_B / s'_A = \exp(2\theta \tan \phi)$

Empirical earth pressure coefficients following one-dimensional strain

Coefficient of earth pressure in 1D plastic compression (normal compression)

$$K_{o,nc} = 1 - \sin \phi_{crit}$$

Coefficient of earth pressure during a 1D unloading-reloading cycle (overconsolidated soil)

$$K_o = K_{o,nc} \left[1 + \frac{(n-1)(n_{max}^\alpha - 1)}{(n_{max} - 1)} \right]$$

where n is current overconsolidation ratio (OCR) defined as $\sigma'_{v,max} / \sigma'_v$

n_{max} is maximum historic OCR defined as $\sigma'_{v,max} / \sigma'_{v,min}$

α is to be taken as $1.2 \sin \phi_{crit}$

Cylindrical cavity expansion

Expansion $\delta A = A - A_o$ caused by increase of pressure $\delta \sigma_c = \sigma_c - \sigma_o$

At radius r : small displacement $\rho = \frac{\delta A}{2\pi r}$

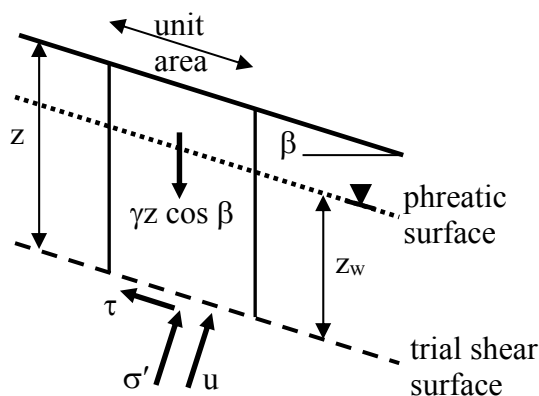
small shear strain $\gamma = \frac{2\rho}{r}$

Radial equilibrium: $r \frac{d\sigma_r}{dr} + \sigma_r - \sigma_\theta = 0$

Elastic expansion (small strains) $\delta \sigma_c = G \frac{\delta A}{A}$

Undrained plastic-elastic expansion $\delta \sigma_c = c_u \left[1 + \ln \frac{G}{c_u} + \ln \frac{\delta A}{A} \right]$

Infinite slope analysis



$$\begin{aligned} u &= \gamma_w z_w \cos^2 \beta \\ \sigma &= \gamma z \cos^2 \beta \\ \sigma' &= (\gamma z - \gamma_w z_w) \cos^2 \beta \\ \tau &= \gamma z \cos \beta \sin \beta \end{aligned}$$

$$\tan \phi_{mob} = \frac{\tau}{\sigma'} = \frac{\tan \beta}{\left(1 - \frac{\gamma_w z_w}{\gamma z}\right)}$$

Shallow foundation design

Tresca soil, with undrained strength s_u

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation for undrained loading (Tresca soil) is:

$$\frac{V_{ult}}{A} = q_f = s_c d_c N_c s_u + \gamma h$$

V_{ult} and A are the ultimate vertical load and the foundation area, respectively. h is the embedment of the foundation base and γ (or γ') is the appropriate density of the overburden.

The exact bearing capacity factor N_c for a plane strain surface foundation (zero embedment) on uniform soil is:

$$N_c = 2 + \pi \quad (\text{Prandtl, 1921})$$

Shape correction factor:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_c = 1 + 0.2 B / L$$

The exact solution for a rough circular foundation ($D = B = L$) is $q_f = 6.05s_u$, hence $s_c = 1.18 \sim 1.2$.

Embedment correction factor:

A fit to Skempton's (1951) embedment correction factors, for an embedment of h , is:

$$d_c = 1 + 0.33 \tan^{-1} (h/B) \quad (\text{or } h/D \text{ for a circular foundation})$$

Combined V-H loading

A curve fit to Green's lower bound plasticity solution for V-H loading is:

$$\text{If } V/V_{ult} > 0.5: \quad \frac{V}{V_{ult}} = \frac{1}{2} + \frac{1}{2} \sqrt{1 - \frac{H}{H_{ult}}} \quad \text{or} \quad \frac{H}{H_{ult}} = 1 - \left(2 \frac{V}{V_{ult}} - 1 \right)^2$$

$$\text{If } V/V_{ult} < 0.5: \quad H = H_{ult} = Bs_u$$

Combined V-H-M loading

With lift-off: combined Green-Meyerhof

$$\text{Without lift-off: } \left(\frac{V}{V_{ult}} \right)^2 + \left[\frac{M}{M_{ult}} \left(1 - 0.3 \frac{H}{H_{ult}} \right) \right]^2 + \left| \left(\frac{H}{H_{ult}} \right)^3 \right| - 1 = 0 \quad (\text{Taiebet \& Carter 2000})$$

Frictional (Coulomb) soil, with friction angle ϕ

Vertical loading

The vertical bearing capacity, q_f , of a shallow foundation under drained loading (Coulomb soil) is:

$$\frac{V_{ult}}{A} = q_f = s_q N_q \sigma'_{v0} + s_\gamma N_\gamma \frac{\gamma' B}{2}$$

The bearing capacity factors N_q and N_γ account for the capacity arising from surcharge and self-weight of the foundation soil respectively. σ'_{v0} is the in situ effective stress acting at the level of the foundation base.

For a strip footing on weightless soil, the exact solution for N_q is:

$$N_q = \tan^2(\pi/4 + \phi/2) e^{(\pi \tan \phi)} \quad (\text{Prandtl 1921})$$

An empirical relationship to estimate N_γ from N_q is (Eurocode 7):

$$N_\gamma = 2(N_q - 1) \tan \phi$$

Curve fits to exact solutions for $N_\gamma = f(\phi)$ are (Davis & Booker 1971):

$$\text{Rough base: } N_\gamma = 0.1054 e^{9.6\phi}$$

$$\text{Smooth base: } N_\gamma = 0.0663 e^{9.3\phi}$$

Shape correction factors:

For a rectangular footing of length L and breadth B (Eurocode 7):

$$s_q = 1 + (B \sin \phi) / L$$

$$s_\gamma = 1 - 0.3 B / L$$

For circular footings take $L = B$.

Combined V-H loading

The Green/Sokolovski lower bound solution gives a V-H failure surface.

Combined V-H-M loading

With lift-off- drained conditions - use Butterfield & Gottardi (1994) failure surface shown above

$$\left[\frac{H/V_{ult}}{t_h} \right]^2 + \left[\frac{M/BV_{ult}}{t_m} \right]^2 + \left[\frac{2C(M/BV_{ult})(H/V_{ult})}{t_h t_m} \right] = \left[\frac{V}{V_{ult}} \left(1 - \frac{V}{V_{ult}} \right) \right]^2$$

$$\text{where } C = \tan \left(\frac{2\rho(t_h - t_m)(t_h + t_m)}{2t_h t_m} \right) \quad (\text{Butterfield \& Gottardi, 1994})$$

Typically, $t_h \sim 0.5$, $t_m \sim 0.4$ and $\rho \sim 15^\circ$. Note that t_h is the friction coefficient, $H/V = \tan \phi$, during sliding.

