Version FC/3

EGT2
ENGINEERING TRIPOS PART IIA

Monday 27 April 2015 14.00 to 15.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data sheet for Question 3 (1 page)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Figure 1(a) shows two thin-walled cross-sections with constant thickness t.

(i) Determine the St. Venant torsion constant of both cross-sections. [20%]

(ii) Explain for each cross-section whether restrained warping torsion is important. [10%]

(b) Figure 1(b) shows an unsymmetrical thin-walled cross-section with constant thickness t.

(i) First compute the principal second moments of area of the cross-section. Subsequently, for a given moment M_x around the x-axis compute the axial stress at point A. [50%]

(ii) For a given shear force S_y, write down the equation for computing the shear flow in the cross-section. No numerical calculations are necessary. [20%]

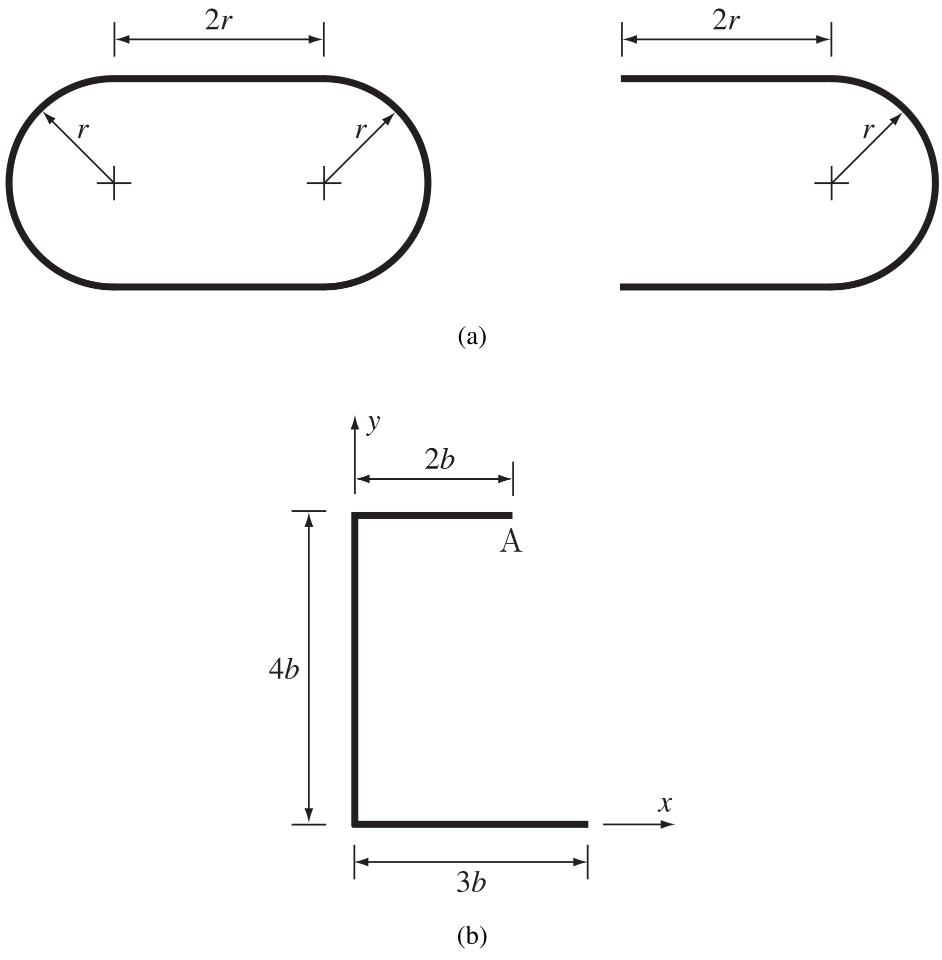


Fig. 1

2 (a) Briefly explain what is meant by warping and explain the role played by resistance to warping in determining the critical moment for lateral-torsional buckling of I-beams. [20%]

(b) Figure 2 shows three light rigid bars of length L with pinned connections. A linear spring of stiffness k forms a diagonal brace. The spring is unstressed when the structure carries no load and adjacent bars are at right angles, as shown. The structure is loaded by vertical and horizontal forces V and H in the directions shown. Answer the following questions using small displacement theory throughout.

(i) Derive an expression for the Total Potential Energy as a function of the sway angle θ. [30%]

(ii) Determine the value of $V = V_{cr}$ at which the system will buckle. [10%]

(iii) For general H and V, derive an expression for the angle θ at equilibrium, and sketch a graph of V versus θ when H is constant. [20%]

(iv) For some fixed value of H, determine the equilibrium angle θ for the cases when $V = 0.4kL$ and $V = -1.5kL$. [20%]

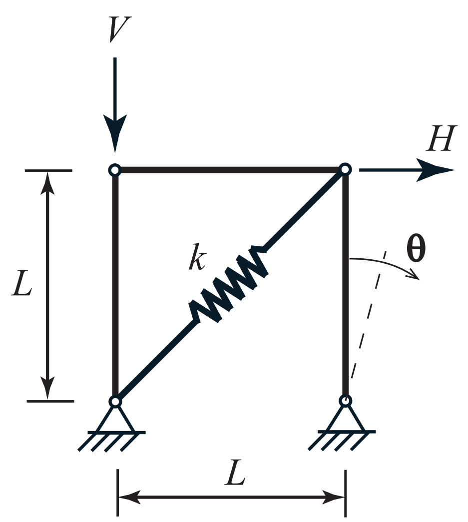


Fig. 2

3 (a) Write down the statement of the reciprocal theorem in form of an equation. Consider a structure of your choice for illustration. [20%]

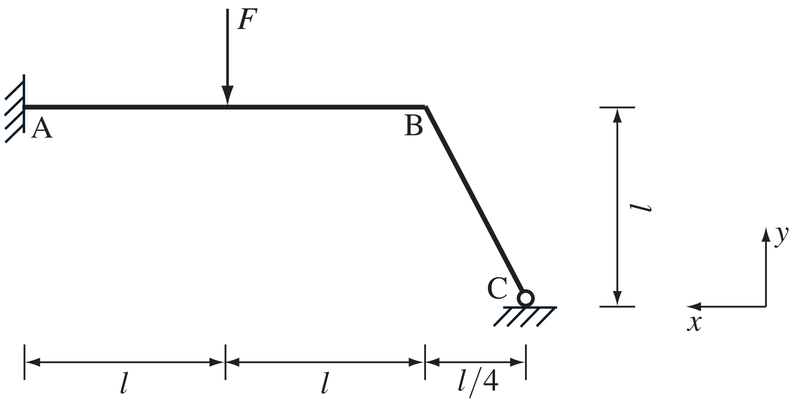
(b) Figure 3(a) shows a frame consisting of two beams that are rigidly connected at B. The flexural stiffness of the two beams is EI and their axial stiffness can be assumed to be infinite. The horizontal beam carries a concentrated vertical load F at its mid-span.

(i) Give the number of degrees of kinematical and statical indeterminacy of the structure. [10%]

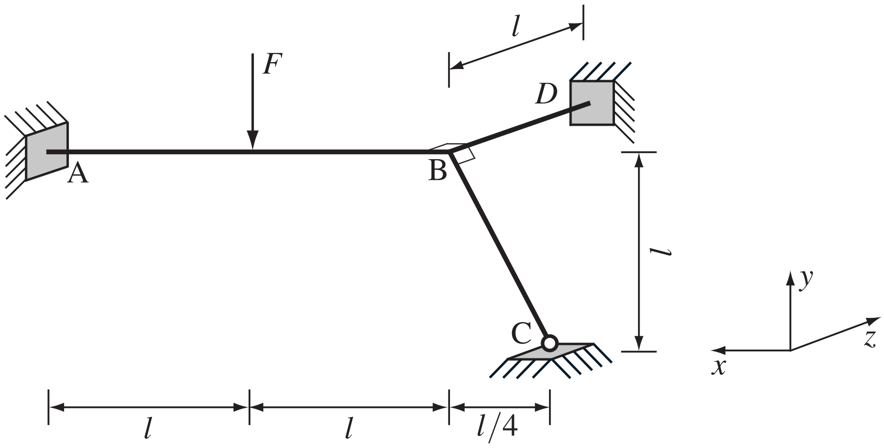
(ii) Calculate the rotation of the connection B. (Datasheet attached) [30%]

(iii) Calculate the bending moment and the shear force at the support A. [20%]

(c) Figure 3(b) shows a three-dimensional frame, which was obtained by propping the two-dimensional frame of Fig. 3(a) in the out-of-plane direction. The torsional stiffness of the propping beam is $GJ = 3EI$. Calculate the rotation of the connection B. [20%]



(a)



(b)

Fig. 3

4 (a) Explain briefly Shanley's resolution of the so-called "Column Paradox", and explain why this presents something of a difficulty for finite element programmes which are built on the Principle of Virtual Displacements. [20%]

(b) The subframe shown in elevation in Fig. 4 consists of a beam BD and two columns AB and BC, all rigidly connected to each other at B. Each member is of length L and has flexural rigidity EI for bending deformations in the plane of the diagram. Buckling out of the plane of the diagram is prevented. All members may be considered to be axially rigid. Boundary conditions are as illustrated and a compressive axial force P is applied at A. The table of the stability functions s and c are provided in Table 1.

(i) Determine, as a function of EI and L, the critical value of P that will cause in-plane elastic instability. [40%]

(ii) Determine the relative magnitudes of the rotations at A, B, C and D as the subframe undergoes elastic buckling, and sketch the buckling mode shape. [20%]

(iii) Explain why the lack of rollers at support D is crucial to the calculation of the elastic critical load. Describe how such a support condition could be achieved in practice in a multi-storey building. [10%]

(iv) Determine the critical value of P that will cause elastic instability if support C were to be a pin whilst support D was on rollers. [10%]

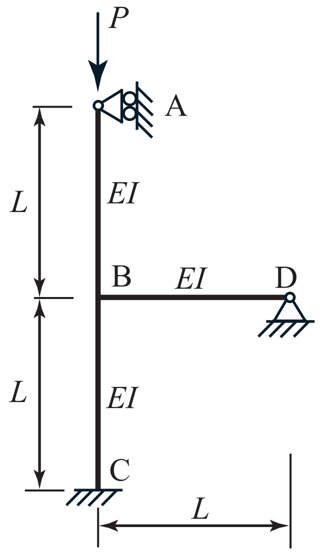


Fig. 4

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

Table 1. s and c stability functions. P is the axial load and P_E is the Euler load.

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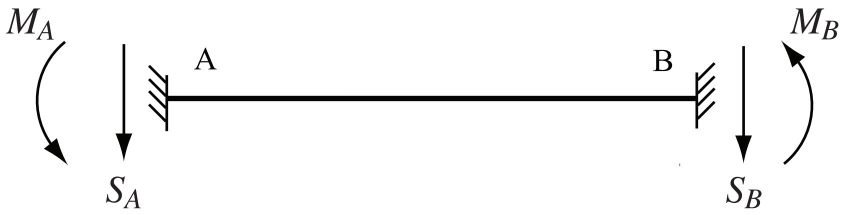
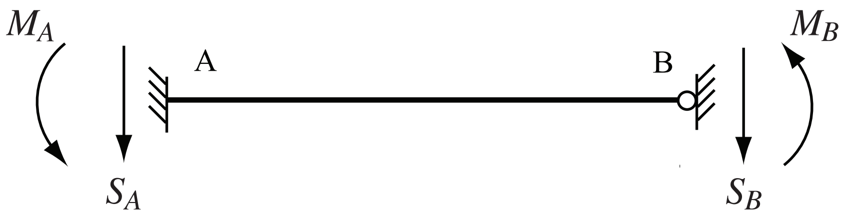
Data Sheet for Question 3: Stiffness Matrices.

Beam type I



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L^3} & -\frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 \\ -\frac{3EI}{L^2} & \frac{3EI}{L} & \frac{3EI}{L^2} & 0 \\ -\frac{3EI}{L^3} & \frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$

Numerical Answers

1. $J = 19.84\, r^3 t,\ J = 2.381\, rt^3,\ I_{xx} = 24.89\, b^3 t,\ I_{yy} = 6.972\, b^3 t,\ I_{xy} = -3.556\, b^3 t,$ $I_{\xi\xi} = 25.57\, b^3 t$, $I_{\eta\eta} = 6.29\, b^3 t$, $\sigma = 0.1245\, M_x/b^2/t$

2. $V_{cr} = 1/2\, kL$, $\theta_{0.4kL} = 10H/k/L$, $\theta_{-1.5kL} = 0.5\, H/k/L$

3. $\theta = 0.0509\, Fl^2/E/I$, $M = 0.301\, Fl$, $S = 0.576\, Fl$, $\theta = 0.0316\, Fl^2/E/I$

4. $P_{cr} = 14.8\, EI/L^2$, $\theta_B = -0.49\, \theta_A$, $\theta_D = 0.24\theta_A$