EGT2
ENGINEERING TRIPOS PART IIA

Monday 18 April 2016 9.30 to 11.00

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Data Sheet for Question 2 (1 page)
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Figure 1(a) shows a thin-walled Z-section with constant thickness t. It is made of material with Young's modulus E and shear modulus G. The Z-section is mounted as a cantilever of length L and is subjected to a constant torque T.

(i) Compute the St Venant torsion constant J of the cross-section. [10%]

(ii) Estimate the twist at the cantilever tip when full warping restraint is provided at the cantilever base. Note that for a Z-section the restrained warping constant is $\Gamma = \frac{d^2}{4} I_{yy}$ and the characteristic length is $\lambda = \sqrt{\frac{E\Gamma}{GJ}}$. Choose the variable d suitably. [20%]

(b) Figure 1(b) shows a thin-walled double channel section with constant thickness t.

(i) Compute the principal second moments of area of the cross-section. [50%]

(ii) For a given shear force S_y, write down the equation for computing the shear flow in the cross-section. No numerical calculations are necessary. [20%]

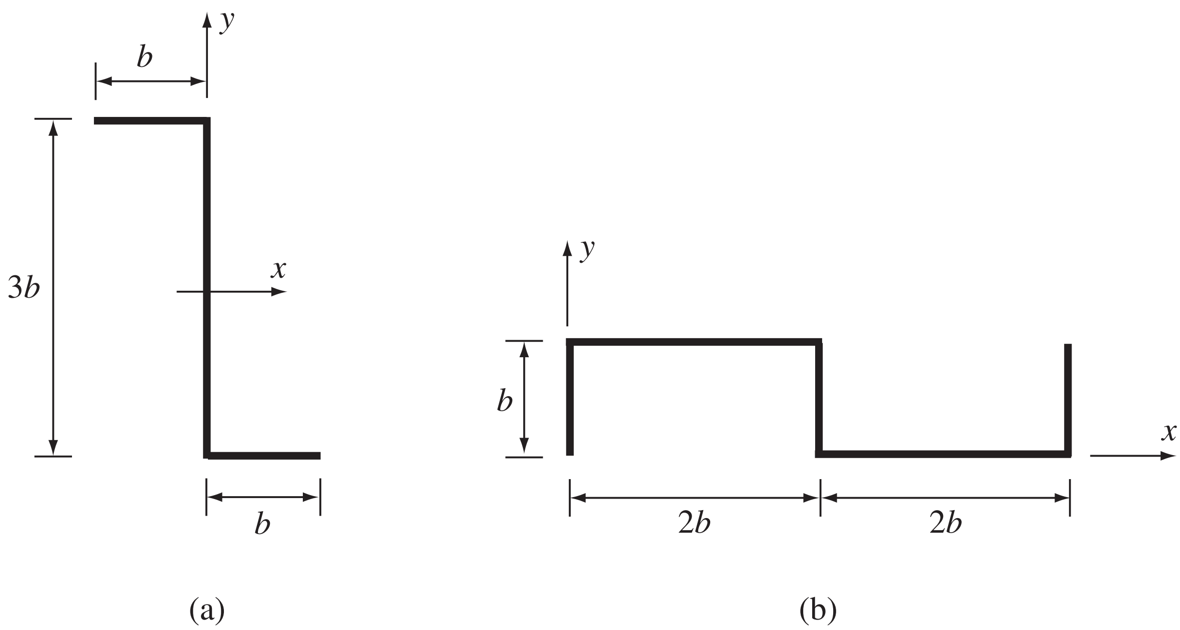

Fig. 1

2 Figure 2 shows the side view of a continuous beam bridge with uniform flexural rigidity and two unequal spans. The beam is simply supported at its two ends and continuous over the middle span. The right span caries a uniformly distributed load w.

(a) Compute the hogging moment over support B using the force method. [30%]

(b) Compute the hogging moment over support B using the displacement method. See attached Data Sheet for element stiffness matrices. [30%]

(c) Use Macaulay's method to produce a set of simultaneous equations that can be used for computing the support reactions. Do not attempt to solve the simultaneous equations obtained. [30%]

(d) How would your equations in (c) be altered for computing the influence line for the sagging moment at midspan D? [10%]

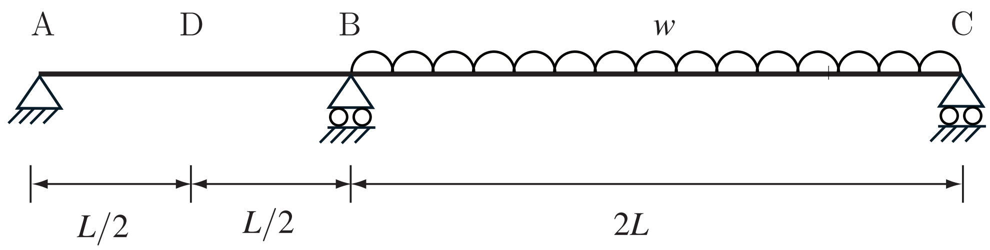

Fig. 2

3 (a) Sketch the patterns of shear stress on the cross-section of a Universal Beam that would be associated with St Venant torsion and with restrained warping torsion. [10%]

(b) The formula for the critical value of equal and opposite end moments that cause lateral-torsional buckling of a beam includes a warping correction factor :

$$\left(1+\frac{\pi^2 E\Gamma}{L^2 GJ}\right)^{1/2}$$

Here, all symbols have their usual meanings, and in particular the warping constant $\Gamma = I_{minor}D_f^2/4$ where D_f is the distance between flange centroids.
Determine the critical value of equal and opposite end-moments that will cause lateral-torsional buckling of a simply-supported steel Universal Beam of length 6 m and of $533 \times 210 \times 101$ section. [40%]

(c) Figure 3 shows five light rods, each of length L and flexural rigidity EI, which are rigidly connected to each other at the joint A. The angle between adjacent rods is 45°. The joint A has a pinned support, and all other supports are pins on rollers as shown. External forces, each of magnitude P, are applied simultaneously at points B and C, each acting towards joint A.
Determine the critical value of P that will cause in-plane elastic instability, and sketch the buckling mode. See Table 1 for stability functions s and c. [50%]

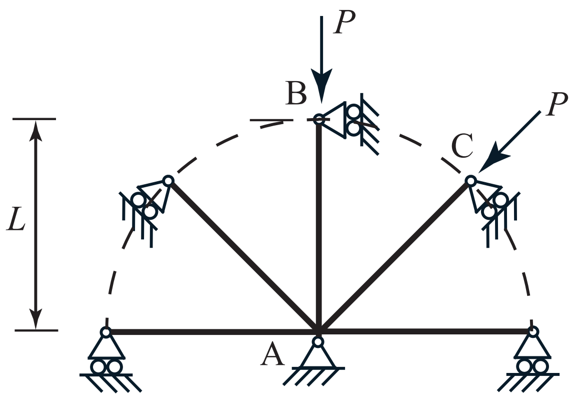

Fig. 3

 (cont.

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	-0.5194	-7.5107
2.4	-1.3006	-3.3703
2.6	-2.2490	-2.2312
2.8	-3.4449	-1.7081
3.0	-5.0320	-1.4157

Table 1. s and c stability functions. P is the axial load and P_E is the Euler load.

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4 (a) Describe the residual stresses that may be generated in a steel Universal Column section as a result of its manufacture by hot-rolling, and briefly explain how these may affect the buckling behaviour of the column. [20%]

(b) State the assumptions that underlie the Rayleigh-Ritz method when applied to estimate the buckling load of a structure. Give a practical example of a structure where these assumptions do not hold. [20%]

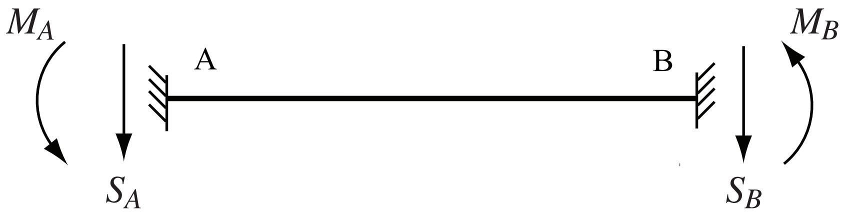
(c) Assuming a parabolic deflected shape of the form $x(L-x)$ for a column of effective length L and flexural rigidity EI (were x is the distance along the beam measured from one end), derive the Rayleigh-Ritz approximation to the critical axial load that will cause elastic flexural instability. Explain briefly why your answer differs from the Euler load, and why it is either higher or lower (as the case may be). [40%]

(d) Explain briefly how the Perry-Robertson formula for the critical load for flexural buckling of a column deals with inelastic behaviour. [20%]

END OF PAPER

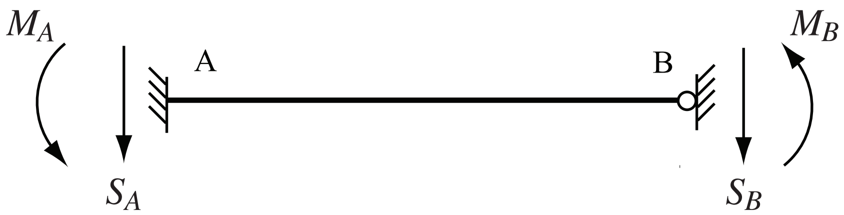
Data Sheet for Question 2: Stiffness Matrices.

Beam type I



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{12EI}{L^3} & -\frac{6EI}{L^2} & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{4EI}{L} & \frac{6EI}{L^2} & \frac{2EI}{L} \\ -\frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{12EI}{L^3} & \frac{6EI}{L^2} \\ -\frac{6EI}{L^2} & \frac{2EI}{L} & \frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$

Beam type II



$$\begin{bmatrix} S_A \\ M_A \\ S_B \\ M_B \end{bmatrix} = \begin{bmatrix} \frac{3EI}{L^3} & -\frac{3EI}{L^2} & -\frac{3EI}{L^3} & 0 \\ -\frac{3EI}{L^2} & \frac{3EI}{L} & \frac{3EI}{L^2} & 0 \\ -\frac{3EI}{L^3} & \frac{3EI}{L^2} & \frac{3EI}{L^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} w_A \\ \phi_A \\ w_B \\ \phi_B \end{bmatrix}$$