EGT2
ENGINEERING TRIPOS PART IIA

Monday 24 April 2017   14.00 to 15.30

Module 3D4

STRUCTURAL ANALYSIS AND STABILITY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper
Graph Paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A bridge ABC, shown in plan view in Fig. 1, is simply supported at A and C where the bending moment is zero but twist is prevented. The supports do not induce any indeterminate horizontal reactions. The bridge consists of two arcs of radius R, each subtending 30°, joined tangentially at B. The bridge has a uniform cross-section throughout its length and carries a uniformly distributed load of intensity w.

(a) By considering the symmetry of the structure

(i) What can you say about the torque at C? [5%]

(ii) What kinematic conditions must apply at B? [10%]

(b) By analysing the segment AB, and applying the appropriate boundary conditions, find the values of T_0 and hence the torque and moment at B. [65%]

(c) Without doing further calculations, describe how the analysis would be modified if the bridge was only loaded in the segment AB. [20%]

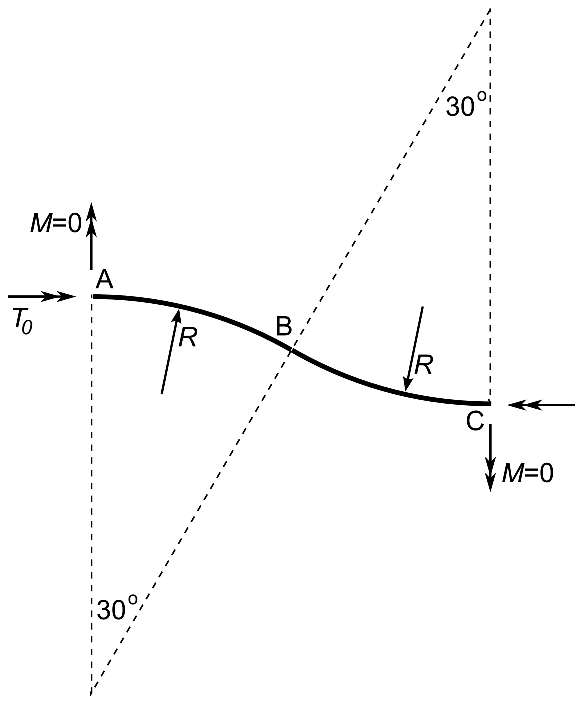

Fig. 1

2 A three-span beam, ABCD is shown Fig. 2. It has a uniform stiffness EI and carries a single load W in the centre of span AB.

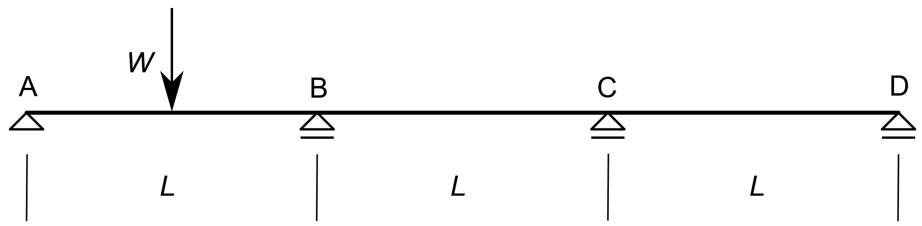

Fig. 2

(a) Derive the equations for a stiffness analysis of the beam taking as your variables the relevant parameters at B and C. Express these equations clearly in matrix form and identify the stiffness matrix. Do not solve the resulting equations. [40%]

(b) Repeat the procedure for a flexibility analysis. [40%]

(c) Solve the flexibility equations and sketch the bending moment diagram for the beam, marking salient values. [20%]

3 Figure 3 shows two versions of a frame structure braced against out-of-plane displacements. In (a) the frame is fully-fixed at A, and pin-jointed at C, while in (b) the horizontal constraint at A has been released. Each member is of length L and has flexural rigidity EI for bending deformations in the plane of the frame.

(a) A vertical load F is applied to frame (a) at B. Estimate the critical load to cause buckling, using the s & c functions given in Table 1, where the stiffness factor s and carry-over factor c are given for a column with Euler buckling load P_E carrying a compressive load P. [40%]

(b) For frame (b), find the 3×3 tangent stiffness matrix, written in terms of the s & c functions, that relates the rotation at B and C and the horizontal displacement at B (θ_B, θ_C and δ_B respectively) to the work-conjugate loads. [60%]

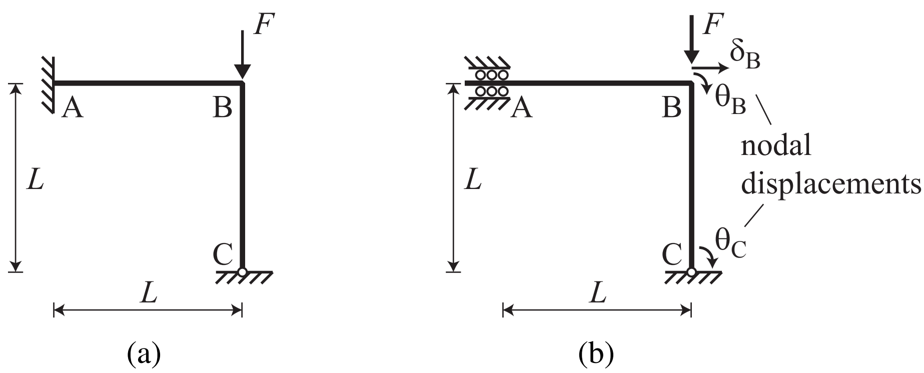

Fig. 3

P/P_E	s	c
0.0	4.0000	0.5000
0.2	3.7297	0.5550
0.4	3.4439	0.6242
0.6	3.1403	0.7136
0.8	2.8159	0.8330
1.0	2.4674	1.0000
1.2	2.0901	1.2487
1.4	1.6782	1.6557

P/P_E	s	c
1.6	1.2240	2.4348
1.8	0.7170	4.4969
2.0	0.1428	24.6841
2.2	−0.5194	−7.5107
2.4	−1.3006	−3.3703
2.6	−2.2490	−2.2312
2.8	−3.4449	−1.7081
3.0	−5.0320	−1.4157

Table 1

4 (a) Figure 4(a) shows three rigid bars of length L arranged in a straight line, connected by pin-joints that constrain any displacements to lie in the plane of the figure. The bars carry a compressive load P. Each of the interior joints is held in place by a spring of stiffness k, which is unstressed in the configuration shown. Assuming small deflections, write down an expression for the total potential energy. Calculate the elastic critical loads and the corresponding buckling modes. [50%]

(b) The system now has the initial imperfection shown in Fig. 4(b). Each joint has an initial displacement of $-\varepsilon L$, where $\varepsilon \ll 1$, when the system is unstressed.

(i) Write down a revised expression for the total potential energy when a compressive load is applied. [30%]

(ii) Sketch the displacements of the joints as an axial load is applied and estimate the elastic critical load. [20%]

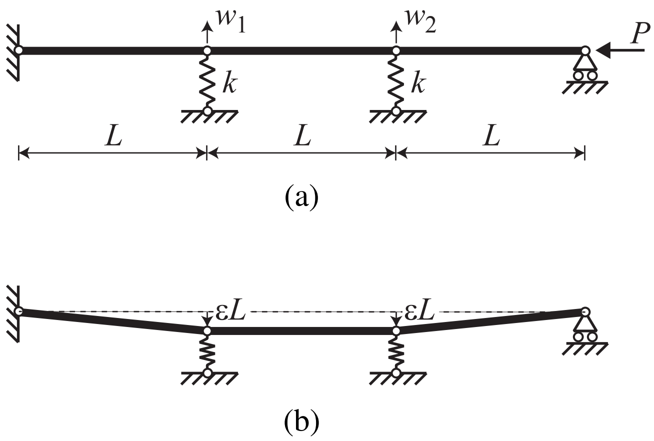

Fig. 4

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Numerical answers

1. (b) $-0.0185wR^2$, $0.0305wR^2$, $-0.137wR^2$
3. (a) $14.5\pi^2 EI/L^2$
4. (a) kL, $[1 \quad -1]$, $3kL$, $[1 \quad 1]$