Version DL/3

EGT2
ENGINEERING TRIPOS PART IIA

Friday 3 May 2024   14.00 to 15.40

Module 3D5

WATER ENGINEERING

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

The values of relevant parameters are listed at the end of the 3D5 data sheet unless otherwise noted in the question.

STATIONERY REQUIREMENTS
Single-sided script paper
Graph paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3D5 Water Engineering data sheet (5 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Identify and describe three major catchment characteristics that might influence its hydrograph. Describe at least two human processes that can alter these characteristics. [25%]

(b) What is an El Nino? Give physical arguments to describe why an El Nino can simultaneously cause a temporary collapse of the fishing industry in Peru and the wheat production in Australia. [25%]

(c) Rain falls uniformly over a catchment for three hours and the excess rainfall is constant over the catchment. The distribution percentages for the outflow hydrograph over six successive 3-hour periods starting from the beginning of the rainfall are 4, 15, 30, 25, 15, 11. Plot the shape of the hydrograph produced by a two-hour excess rainfall of uniform distribution over this catchment that might occur in future. [50%]

2 (a) An irrigation channel is of a symmetric trapezoidal cross-sectional shape, with a bed width of 3 m and a side-slope angle of 30^{o} relative to the horizontal. The Manning roughness coefficient is 0.015 s $m^{-1/3}$. When the flow rate is 50 m^3 s^{-1}, the uniform water depth is measured to be 2 m.

(i) Determine whether the flow is subcritical or supercritical. [30%]

(ii) Estimate the bed slope of the channel. [30%]

(b) A wide river flows to the sea. Initially the water is still, with a depth of 4 m. Then, the sea level begins to drop at 1 m hr^{-1}. It can be assumed that the river bed is flat and frictionless. After one hour of the sea level falling, at what distance upriver from the river mouth will the water depth have dropped by 0.5 m? [40%]

3 Flow in a wide river can be assumed to be uniform, with depth 2 m and velocity 1 m s^{-1}. The bed slope is 4×10^{-4}. The grain size of the bed material is 0.3 mm. Take the grain-related roughness height to be twice the grain size.

(a) Estimate the total shear stress, grain-related shear stress and bedform-related shear stress acting on the riverbed. [20%]

(b) Estimate the bedform-related roughness height based on the information given above. [10%]

(c) Using the near-bed sediment concentration formula of Zyserman and Fredsøe, estimate the suspended sediment concentration in kg m^{-3} at mid depth of the river. [40%]

(d) A factory discharges pollutant water at a constant rate from an outfall located at one bank of the river. The pollutant concentration 50 m downstream of the outfall and 1 m away from the bank is measured to be 0.01 g $litre^{-1}$. Estimate the mass of the pollutant discharged by the factory each day. [30%]

4 (a) Derive from first principles the Chezy formula for uniform flows. [20%]

(b) A centrifugal pump when running at 750 rpm gives the following relationship between the discharge rate Q, head H and efficiency:

Q (litre s^{-1})	0	5	10	15	20	25
H (m)	32.5	32	31	29	22	6
Efficiency (%)	-	59	74	83	79	70

The pump is connected to a pipe system which discharges water from a sump to a water tower. The water level difference between the sump and the water tower is 20 m.

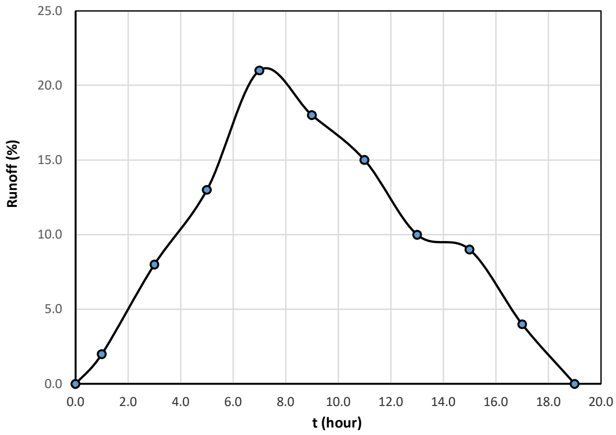
(i) The total mechanical energy head loss (in m), including friction loss and local loss, in the pipe system can be expressed to be $0.02Q^2$, where Q is in litre s^{-1}. Calculate the discharge rate and the pump input power. [30%]

(ii) It is expected that future encrustation in the pipe will reduce the pipe diameter and increase roughness so that the total mechanical energy head loss (in m) in the system will become $0.03Q^2$, where Q is in litre s^{-1}. Estimate the new pump speed and new input power required to maintain the same discharge rate as in part 4 (b) (i). [50%]

END OF PAPER

Answers

Q1 (c)



Q2 (a) (i) Supercritical, (ii) $S_b = 0.0027$

Q2 (b) Around 9090 m

Q3 (a) Total: 7.85 Pa; Grain-related: 1.44 Pa; Bedform-related: 6.41 Pa

Q3 (b) 0.26 m

Q3 (c) 0.0026 kg/m^3

Q3 (d) around 4234 kg

Q4 (b) (i) 17.6 litre/s, 5500 W; (ii) 780 rpm, 6120 W

UNIVERSITY OF CAMBRIDGE

Engineering Tripos Part IIA   Third Year

Module 3D5: Water Engineering
Data Book (SI units [m, kg, s] unless otherwise noted)

Hydrology

Horton's infiltration model (*f*-capacity) $f = f_c + (f_0 - f_c)e^{-K_f t}$

$$\int_{t_1}^{t_2} f \cdot dt = f_c(t_2 - t_1) - \frac{1}{K_f}(f_0 - f_c)\left(e^{-K_f t_2} - e^{-K_f t_1}\right)$$

Rational method $Q = CiA$

Boundary Layer

For fully developed boundary layer flow with a free surface (uniform flow in a very wide channel):

Eddy viscosity coefficient $\nu_t = \kappa u_* z\left(1 - \frac{z}{h}\right)$

Velocity in hydraulically smooth regime ($u_* k_s / \nu < 5$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(9.0\frac{z u_*}{\nu}\right)$

Velocity in hydraulically rough regime ($u_* k_s / \nu > 70$) $\frac{\bar{u}(z)}{u_*} = \frac{1}{\kappa}\ln\left(\frac{30.0 z}{k_s}\right)$

Open Channel Flow

Chézy coefficient in large Reynolds-number flows $C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right)$

Manning formula $C = \frac{1}{n} \cdot R_h^{1/6}$

Bed shear stress $\tau_b = \rho g R_h S_f = \frac{C_f}{2}\rho \cdot U^2 = \frac{\lambda}{8}\rho \cdot U^2 = \frac{g}{C^2}\rho \cdot U^2 = \frac{g \cdot n^2}{R_h^{1/3}}\rho \cdot U^2 = \rho \cdot u_*^2$

Froude number $Fr = \frac{U}{\sqrt{g A/B}}$

Uniform flows (Chézy formula) $U = C\sqrt{R_h S_b}$

Hydraulic jumps in rectangular channels $U_1 U_2 = g\frac{h_1 + h_2}{2}$

Gradually varied steady flows: $\frac{d}{dx}\left(h + \frac{U^2}{2g}\right) = S_b - S_f$

or $\frac{dh}{dx} = \frac{S_b - S_f}{1 - Fr^2} = \frac{S_b - \frac{U^2}{C^2 \cdot R_h}}{1 - Fr^2} = \frac{S_b - \frac{n^2 \cdot U^2}{R_h^{4/3}}}{1 - Fr^2}$

Characteristics for unsteady flows in rectangular channels:

$$\frac{d}{dt}\left(U + 2\sqrt{gh}\right) = g\left(S_b - S_f\right) \text{ along } \frac{dx}{dt} = U + \sqrt{gh}$$

$$\frac{d}{dt}\left(U - 2\sqrt{gh}\right) = g\left(S_b - S_f\right) \text{ along } \frac{dx}{dt} = U - \sqrt{gh}$$

Pollutant Transport

Mixing coefficients in wide rectangular channels: $D_{tx} = D_{ty} = 0.15hu_*$, $D_{tz} = 0.067hu_*$, $D_L = 5.86hu_*$

For instantaneous release from origin at $t = 0$ in uniform flows along x direction:

One-dimensional $$\bar{c}(x,t) = \frac{M/A}{(4\pi t)^{1/2}\sqrt{D_x}} \exp\left(-\frac{(x-Ut)^2}{4D_x t}\right)$$

Two-dimensional $$\bar{c}(x,y,t) = \frac{M/h}{4\pi t\sqrt{D_x D_y}} \exp\left(-\frac{(x-Ut)^2}{4D_x t} - \frac{y^2}{4D_y t}\right)$$

Three-dimensional $$\bar{c}(x,y,z,t) = \frac{M}{(4\pi t)^{3/2}\sqrt{D_x D_y D_z}} \exp\left(-\frac{(x-Ut)^2}{4D_x t} - \frac{y^2}{4D_y t} - \frac{z^2}{4D_z t}\right)$$

For continuous release from origin in uniform flows along x direction:

Two-dimensional $$\bar{c}(x,y) = \frac{\dot{M}/h}{U\sqrt{4\pi \frac{x}{U} D_y}} \exp\left(-\frac{y^2}{4D_y\, x/U}\right)$$

Three-dimensional $$\bar{c}(x,y,z) = \frac{\dot{M}}{4\pi x\sqrt{D_y D_z}} \exp\left(-\frac{y^2}{4D_y\, x/U} - \frac{z^2}{4D_z\, x/U}\right)$$

Sediment Transport

Definitions of Shields parameter, non-dimensional grain size and transport stage parameter:

$$\theta = \frac{\tau_b}{g(\rho_s - \rho)d}, \quad d_* = d \cdot \left(\frac{g(s-1)}{\nu^2}\right)^{1/3}, \quad T = \frac{\tau_b' - \tau_{bc}}{\tau_{bc}} = \frac{\theta' - \theta_c}{\theta_c}$$

Critical Shields parameter $$\theta_c = \frac{0.30}{1 + 1.2d_*} + 0.055\left[1 - \exp(-0.02d_*)\right]$$

Fall velocity $$w_s = \frac{\nu}{d}\left[\sqrt{10.36^2 + 1.049 \cdot d_*^{\,3}} - 10.36\right]$$

Shear stress partition: $$C' = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s'}\right), \quad \tau_b' = \rho g\frac{U^2}{C'^2}$$

$$C = 7.8\ln\left(\frac{12.0 \cdot R_h}{k_s}\right), \quad \tau_b = \rho g\frac{U^2}{C^2}$$

Volumetric bedload transport rate per unit width:

Meyer-Peter and Müller $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 8\left[\left(\frac{C}{C'}\right)^{1.5}\theta - 0.047\right]^{1.5}$$

van Rijn $$\frac{q_b}{\sqrt{g(s-1)\cdot d^3}} = 0.053\frac{T^{2.1}}{d_*^{\,0.3}}$$

Rouse profile of suspended sediment concentration $$\frac{\bar{c}(z)}{\bar{c}(a)} = \left(\frac{h-z}{z} \cdot \frac{a}{h-a} \right)^{\frac{w_s}{\kappa u_*}}$$

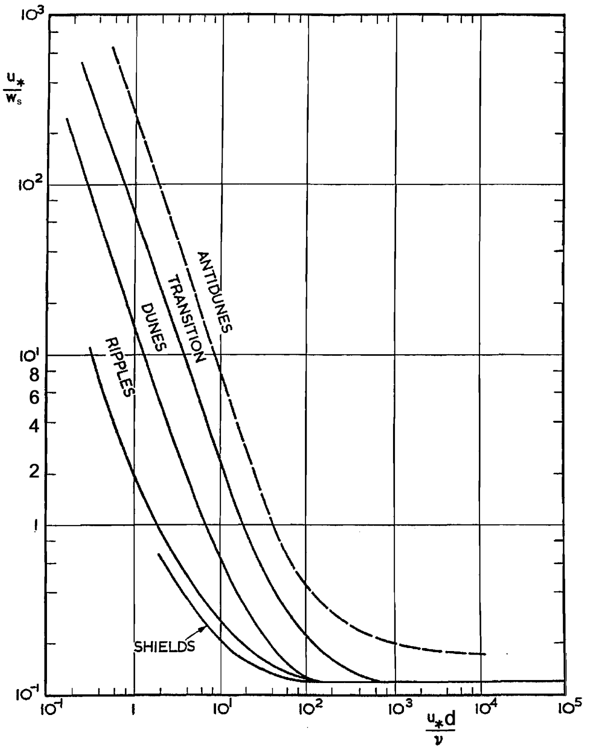
Reference volumetric concentration close to the bed:

Zyserman and Fredsøe $$\bar{c}(2d) = \frac{0.331 \cdot (\theta' - 0.045)^{1.75}}{1 + 0.72 \cdot (\theta' - 0.045)^{1.75}}$$

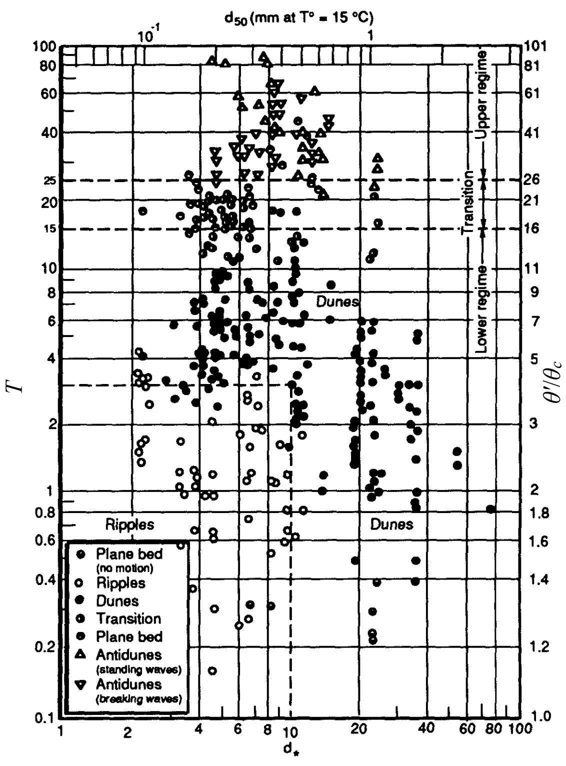
van Rijn $$\bar{c}(a) = 0.015 \frac{d \cdot T^{1.5}}{a \cdot d_*^{0.3}}$$

Suspended load per unit width $$q_s = \int_a^h \bar{c}(z)\bar{u}(z)dz = 11.6 \cdot u_* \cdot \bar{c}(a) \cdot a \cdot \left[I_1 \ln\left(\frac{30h}{k_s}\right) + I_2 \right]$$

a/h	$w_s/(\kappa u_*) = 0.2$		$w_s/(\kappa u_*) = 0.6$		$w_s/(\kappa u_*) = 1.0$		$w_s/(\kappa u_*) = 1.5$	
	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$	I_1	$-I_2$
0.02	5.003	5.960	1.527	2.687	0.646	1.448	0.310	0.873
0.01	8.892	11.20	2.174	4.254	0.788	2.107	0.341	1.146
0.005	15.67	20.47	3.033	6.448	0.934	2.837	0.366	1.431
0.004	18.77	24.73	3.364	7.318	0.981	3.094	0.372	1.525
0.003	23.71	31.53	3.838	8.579	1.042	3.444	0.379	1.647
0.002	32.88	44.23	4.608	10.65	1.129	3.967	0.389	1.819
0.001	57.46	78.30	6.247	15.17	1.277	4.944	0.401	2.117
0.0005	100.2	137.7	8.413	21.26	1.426	6.027	0.409	2.413
0.0001	363.9	504.9	16.50	44.53	1.773	8.947	0.422	3.113



Liu (1957)



Van Rijn (1984)

Pipeline and Pump

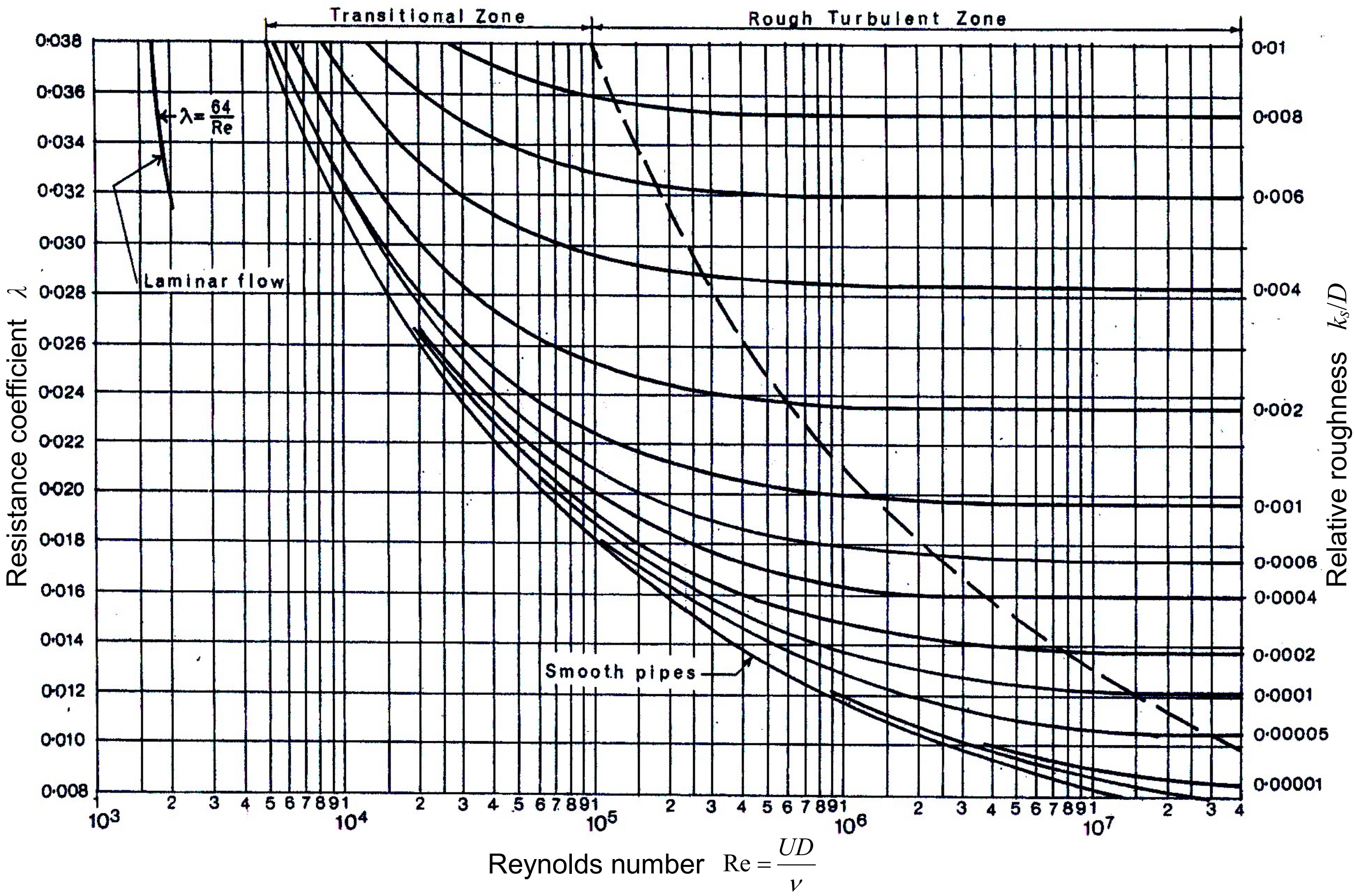
Darcy-Weisbach Equation $\quad H_f = \lambda \dfrac{L}{D}\dfrac{U^2}{2g}$

Colebrook-White formula $\quad \dfrac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\dfrac{k_s}{3.7D} + \dfrac{2.51}{\text{Re}\sqrt{\lambda}}\right)$ with $\text{Re} = \dfrac{UD}{\nu}$

Power consumption $\quad P_p = \rho g Q_p H_p / \eta$

Non-dimensional groups $\quad \dfrac{Q_p}{N_p \cdot D_p{}^3}, \; \dfrac{gH_p}{N_p{}^2 \cdot D_p{}^2}, \; \dfrac{P_p}{\rho \cdot N_p{}^3 \cdot D_p{}^5}$

Specific speed $\quad N_S = \dfrac{N_p \cdot Q_p{}^{1/2}}{g^{3/4} \cdot H_p{}^{3/4}}$



Symbols

A	area
B	channel width at the water surface
C	runoff coefficient or Chézy coefficient
C_f	shear stress coefficient

c	concentration
D	pipeline or pump diameter
D_L	longitudinal dispersion coefficient
D_x,D_y,D_z	diffusion coefficients in x, y and z directions respectively
D_{tx},D_{ty},D_{tz}	turbulent diffusion coefficients in x, y, and z directions respectively
d	particle diameter
d_*	dimensionless particle diameter
F	force
Fr	Froude number
f	infiltration capacity
f_0	initial infiltration capacity
f_c	equilibrium infiltration capacity
g	gravitational acceleration (= 9.81 m s^{-2})
H	head
h	water depth
i	rainfall intensity
K_f	rate of decrease of f capacity
k_s	roughness height, also called equivalent or Nikuradse's sand roughness height
M	amount of the pollutant released
$\dot{M}$	rate of the pollutant release
N	rotational speed
P	power
Q	discharge
q_b	bedload sediment transport rate
R_h	hydraulic radius
S_b	bed slope
S_f	slope of the total energy line
s	specific gravity, also called relative density or density ratio (= 2.65)
T	transport-stage parameter
t	time
U	mean velocity
u_*	shear velocity
w_s	fall velocity
x,y,z	spatial coordinates
θ	Shields parameter
θ_c	critical Shields parameter
η	efficiency
κ	von Karman constant (= 0.4)
λ	Darcy-Weisbach friction factor
ν	kinematic viscosity coefficient of water (= 10^{-6} m^2 s^{-1})
ν_t	eddy viscosity coefficient
ρ	density of water (= 1000 kg m^{-3})
ρ_s	density of sediment (= 2650 kg m^{-3})
τ_b	bed shear stress
τ_{bc}	threshold bed shear stress for particle motion
$\overline{}$	Reynolds-averaged value
$'$	effective shear-stress component, also called grain-related shear-stress component
$_p$	pump

D Liang, Oct 2017