

Version HJ/3

EGT2: IIA
ENGINEERING TRIPOS PART IIA

Tuesday 5 May 2015 9:30 to 11

Module 3E3

MODELLING RISK

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONARY REQUIREMENTS

Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM

CUED approved calculator allowed

Attachment: 3E3 Modelling Risk data sheet (3 pages).

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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1 (a) A car rental agency has offices in Birmingham, Cambridge, Exeter, and London. The agency allows one- and two-way rentals, so that cars rented in one location may end up in another. Statistics shows that at the end of each week 70% of all rentals are two-way. As for the one-way rentals: From Birmingham, 20% go to Cambridge, 60% go to Exeter, and the rest go to London; from Cambridge, 40% go to London and 60% go to Exeter; from Exeter, 50% go to London and the rest go to Cambridge; and from London, 80% go to Exeter, 10% go to Cambridge, and 10% go to Birmingham.

(i) Express the situation as a Markov chain and specify the corresponding transition matrix. [15%]

(ii) If the agency starts the week with 100 cars in each location, what will the distribution be like in two weeks? [15%]

(iii) If each location is designed to handle a maximum of 110 cars, would there be a long run space availability problem in any of the locations? [15%]

(iv) Calculate the average number of weeks that elapse for a car that starts at Cambridge and returns to Cambridge for the first time. [15%]

(b) Labour needs in construction projects can be met through hiring and firing of workers. Both activities incur costs. The goal is to minimize the total cost of labour needed for the project. Assume that the duration of the project is N weeks and that the minimum labour force required in week n is b_n workers. The model assumes that an additional cost is incurred if a week's workforce exceeds the minimum requirement or if additional hiring takes place in a week. For simplicity, no cost is incurred when firing takes place. The cost of maintaining a workforce x_n larger than the minimum b_n in week n incurs a unit excess cost of C_1 . If $x_n > x_{n-1}$, hiring occurs at the additional cost of C_2 per person.

(i) Define relevant parameters, variables and functions and formulate the optimality equations. [20%]

(c) You are given the Summary Outputs of two different multiple linear regression models that are generated using the Excel regression function for the same business problem. The Summary Outputs explore the relationship between a performance measure and several explanatory variables. Discuss how you would assess the relative strengths and weaknesses of each model. [20%]

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2 (a) Business Publishing received a manuscript for a new college management textbook. The editor of Business Publishing is familiar with the manuscript and estimated a 0.65 probability that the textbook will be successful. If successful, a profit of £75,000 will be realized. If Business Publishing decides to publish the textbook and it is unsuccessful, a loss of £150,000 will occur.

(i) Construct a decision tree for Business Publishing. [10%]

(ii) What is the optimal decision for Business Publishing, and what is the expected profit for this project? [10%]

Before making the decision of accepting or rejecting the manuscript, the editor is considering sending the manuscript out for a review. A review process provides either a favourable (F) or unfavourable (U) evaluation of the manuscript. The past experience with the review process suggests probabilities $P(F) = 0.7$ and $P(U) = 0.3$. If the review is favourable, then the textbook will be successful with a 0.75 probability and will be unsuccessful with a 0.25 probability. If the review is unfavourable, then the textbook will be successful with a 0.417 probability and will be unsuccessful with a 0.583 probability.

(iii) Construct a new decision tree for Business Publishing. [10%]

(iv) What is the new optimal decision for Business Publishing, and what is the new expected profit for this project? [10%]

(v) If the cost for the manuscript review is £5,000, what is your recommendation about acceptance or rejection? What is the maximum that Business Publishing should be willing to pay for a manuscript review? Support your answers with detailed analysis. [10%]

(b) For the M/M/1 queue, let λ and μ be the arrival rate and service rate respectively. Let p_n be the steady-state probability that there are n customers in the system. It can be proved that $p_n = c_n p_0$, where $c_n = (\lambda / \mu)^n$.

(i) Derive closed-form solutions for L_q (the average queue length), L (the average number of customers in the system), W_q (the average waiting time in the queue), and W (the average time in the system). [30%]

(c) Specify the mathematical equations for both the simple exponential smoothing model and the Winters multiplicative exponential smoothing model. Explain when you would prefer to use the Winters multiplicative exponential smoothing model over the simple exponential smoothing model. [20%]

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3 (a) Consider a Markov chain with transition probability P_{ij} from state i to state j . Let H_{ij} be the time of the first passage from state i to state j and $E(H_{ij})$ be the expected value of H_{ij} .

(i) Derive the following equation for $E(H_{ij})$:

$$E(H_{ij}) = 1 + \sum_{k \neq j} E(H_{kj})P_{ik}$$

[30%]

(b) A shop uses 10 identical machines. Each machine breaks down once every 7 hours on average. It takes half an hour on average to repair a broken machine. Both the breakdown and repair processes follow the Poisson distribution.

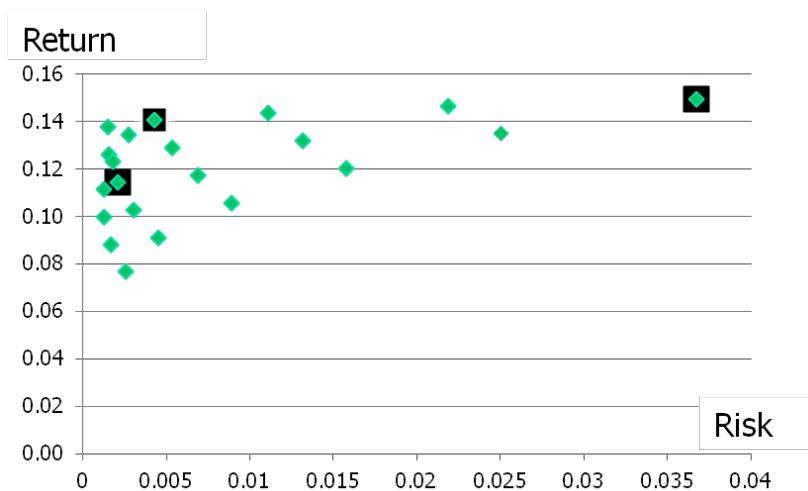
(i) Calculate the average number of broken machines at any point in time when the number of repair persons is equal to 1 and 2, respectively. [20%]

(c) Use calculations and a line chart to illustrate how the width of the 95% confidence interval for the population mean changes with the change of the sample size. What managerial conclusions can you draw from the line chart? [15%]

(d) (i) For simple linear regression, define the R -square statistic mathematically and explain its meaning in words. [10%]

(ii) What are the underlying assumptions for simple linear regression? [10%]

(e) Portfolios generated from three stocks are shown in the risk-return scatter diagram below. Discuss and compare the three portfolios highlighted in large squares. [15%]



END OF PAPER