Version RF/2

EGT2: IIA
ENGINEERING TRIPOS PART IIA

Thursday 28 April 2016 9:30 to 11:00

Module 3E3

MODELLING RISK

Answer not more than ***two*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONARY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 3E3 Modelling Risk data sheet (3 pages).

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

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1 (a) A store is selling television. The demand per week and probability of demand are as follows:

Demand per week (television)	0	1	2	3	4
Probability	0.15	0.2	0.35	0.25	0.05

Unmet demand is lost. The store uses a periodic (s, S) policy: at the end of each weekend, it places an order for delivery at the beginning of the following week if the inventory level drops below 3 televisions (s = 3) and the new replenishment always brings the stock back to 5 televisions (S = 5).

(i) Express the situation as a Markov chain and specify the corresponding transition matrix whose states are numbers of televisions at the beginning of the week. [15%]

(ii) Suppose that the week starts with 4 televisions. Determine the probability that an order will be placed at the end of two weeks. [15%]

(iii) Determine the steady-state probability distribution. [15%]

(iv) If the fixed cost of placing an order is £10, the holding cost per television per week is £0.5, and the penalty cost per unit shortage per week is £2, determine the expected cost per week. [15%]

(b) Cambridge Manufacturing produces two products. The daily capacity of the manufacturing process is 430 minutes. Product 1 requires 2 minutes per unit, and product 2 requires 1 minute per unit. The daily demand for product 1 is equal to infinity, and the maximum daily demand for product 2 is 230 units. The unit profit of product 1 is £2 and the unit profit of product 2 is £5.

(i) Define relevant parameters, variables and functions and formulate the optimality equations for the dynamic programming model in which stage n corresponds to product n. [15%]

(ii) Find the optimal solution for the dynamic programming model. [15%]

(c) Define the continuous (s, S) inventory policy and its relationship with the newsvendor problem. [10%]

2 (a) Farmer Johns can plant either corn or wheat. The probabilities that the next harvest's prices will go up, stay the same, or go down are 0.25, 0.30, and 0.45, respectively. If prices go up, the corn crop will net £30,000 and the wheat crop will net £10,000. If prices remain unchanged, Johns will break even. However, if prices go down, the corn and wheat crops will make losses of £35,000 and £5,000, respectively. Farmer Johns has the additional option of using the land as a grazing range, in which case a payoff of £7,500 is guaranteed.

Farmer Johns has decided to secure additional information from a broker regarding the degree of stability of future prices before Johns makes his decisions. The broker's assessment of ''favourable'' and ''unfavourable'' is described by the following conditional probabilities: $P\{a_1|s_1\}=0.85$, $P\{a_1|s_2\}=0.50$, $P\{a_1|s_3\}=0.15$, $P\{a_2|s_1\}=0.15$, $P\{a_2|s_2\}=0.50$ and $P\{a_2|s_3\}=0.85$, where a_1 and a_2 represent the ''favourable'' and ''unfavourable'' assessments, and s_1, s_2 and s_3 represent ''up'', ''same'' and ''down'' changes in the future prices, respectively.

(i) Construct a decision tree for Farmer Johns. [15%]

(ii) Help Farmer Johns to find the optimal decisions with detailed calculations. [20%]

(b) Consider a queue model with a single server, where only one customer is allowed in the system. Customers who arrive and find the facility busy never return. Assume that the arrival distribution is Poisson with mean of λ and the service time is exponential with the mean service rate of μ.

(i) Set up the transition diagram, and determine the balance equations. [15%]

(ii) Determine the steady-state probabilities. [5%]

(iii) Determine the average number of customers in the system, the average queue length, the average waiting time in the system, the average waiting time in the queue, and the arrival rate for the customers who are served in the system. [15%]

(c) Consider a Markov chain with the following transition matrix:

$$\begin{bmatrix} 0.4 & 0 & 0.6 \\ 0.3 & 0.3 & 0.4 \\ 0 & 0.5 & 0.5 \end{bmatrix}$$

(i) Define the concept of classes and find all classes for the Markov Chain. [10%]

(ii) Define the period for a state and derive the period for all states of the Markov Chain. [10%]

(iii) Define the expected first passage time and describe its relationship with the steady-state probability distribution. [10%]

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3 (a) Consider a one-period newsvendor problem for which the demand D is stochastic with a continuous cumulative distribution F. The unit order cost for the product is £c and the unit revenue for a sold product is £r. Assume there is no penalty for shortages and there is no salvage value for unsold products.

(i) Mathematically, develop the optimisation problem for the newsvendor problem. [10%]

(ii) Derive the optimal solution for the newsvendor problem. [15%]

(b) With regard to linear regression theory:

(i) Consider a multiple regression problem. In words and mathematically, define the correlation coefficient and the R-square statistic. In words, define the coefficient/ slope for an independent variable. [15%]

(ii) For a simple regression problem, discuss the relationships among the correlation coefficient, the R-square statistic and the coefficient/slope for an independent variable. [10%]

(iii) Discuss how you would assess performance of a multiple regression equation. [15%]

(c) Mathematically define Winter's additive exponential smoothing method and compare it with Winter's multiplicative exponential smoothing method. [10%]

(d) Define and discuss the concept of market portfolio. [10%]

(e) Define and discuss the Capital Asset Pricing Model (CAPM). [15%]

END OF PAPER

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ENGINEERING TRIPOS PART IIA
xxx xxx xxx xxx, Module 3E3, Questions 1-3.

SPECIAL DATA SHEET

Standard errors

$$STEM = \frac{\sigma}{\sqrt{n}} \approx \frac{s}{\sqrt{n}},$$

where STEM represents the standard error for the sample mean and it can be used for constructing confidence intervals and prediction intervals in regression analysis.

Covariance, Correlation and Regression

Consider data pairs (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n).
Let m_X and m_Y denote the respective means of the X and Y data.
Let s_X and s_Y denote the respective standard deviations of the X and Y data.
Covariance between X and Y is given by

$$\mathrm{cov}(\mathrm{X},\mathrm{Y}) = \frac{\sum_{i=1}^{n}(\mathrm{X}_i - m_\mathrm{X})(\mathrm{Y}_i - m_\mathrm{Y})}{n} = \frac{\sum_{i=1}^{n}\mathrm{X}_i\mathrm{Y}_i}{n} - m_\mathrm{X} m_\mathrm{Y}$$

The correlation coefficient between X and Y is given by

$$\mathrm{correl}(\mathrm{X},\mathrm{Y}) = r = \frac{\mathrm{cov}(\mathrm{X},\mathrm{Y})}{s_\mathrm{X} s_\mathrm{Y}}.$$

The line of best fit is given by

$$\mathrm{Y} - m_\mathrm{Y} = \frac{r s_\mathrm{Y}}{s_\mathrm{X}}(\mathrm{X} - m_\mathrm{X}).$$

Variance of a portfolio

Consider three random variables x, y and z with means m_x, m_y, and m_z, respectively; variances Var(x), Var(y), and Var(z), respectively; and covariance between x and y, for example, given by the formula above. Given any numbers α_x, α_y, α_z, let $v = \alpha_x x + \alpha_y y + \alpha_z z$. Then the variance of v is given by

$$\begin{aligned}\mathrm{Var}(v) &= \alpha_x^{\,2}\mathrm{Var}(x) + \alpha_y^{\,2}\mathrm{Var}(y) + \alpha_z^{\,2}\mathrm{Var}(z) \\ &\quad + 2\big(\alpha_x\alpha_y\mathrm{cov}(x,y) + \alpha_y\alpha_z\mathrm{cov}(y,z) + \alpha_x\alpha_z\mathrm{cov}(x,z)\big)\end{aligned}$$

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Time Series Forecasting (Winters' multiplicative smoothing method)

$$E_t = \alpha \frac{X_t}{S_{t-c}} + (1-\alpha)(E_{t-1} + T_{t-1})$$

$$T_t = \beta(E_t - E_{t-1}) + (1-\beta)T_{t-1}$$

$$S_t = \gamma \frac{X_t}{E_t} + (1-\gamma)S_{t-c}$$

$$F_{t+k} = (E_t + kT_t)S_{t+k-c}$$

Markov Chains (calculate probabilities for first passage time and expected first passage times)

$$f_{ij}(1) = P_{ij}$$

$$\vdots$$

$$f_{ij}(n) = P_{ij}^{(n)} - f_{ij}(1)P_{jj}^{(n-1)} - \ldots - f_{ij}(n-1)P_{jj}^{(1)}.$$

$$E(H_{ij}) = 1 + \sum_{k \neq j} E(H_{kj})P_{ik}, \forall i.$$

Queueing Theory (Poisson distribution, exponential distribution, performance metrics for the M/M/s queue, the M/M/1 queue is a special case of the M/M/s queue)

$$P(X = k) = \frac{(\lambda t)^k e^{-\lambda t}}{k!}, \quad k = 0,1,\ldots$$

$$P(X \leq t) = 1 - e^{-\mu t}, \quad \forall t \geq 0.$$

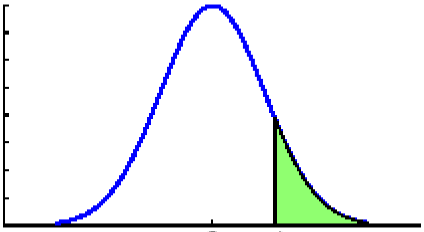
$$p_0 = \frac{1}{\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!}\left(\frac{s\mu}{s\mu - \lambda}\right)}$$

$$p_n = \begin{cases} \frac{(\lambda/\mu)^n}{n!} p_0 & \text{if } 0 \leq n \leq s \\ \frac{(\lambda/\mu)^n}{s! s^{n-s}} p_0 & \text{if } n \geq s \end{cases}$$

$$L_q = \left(\frac{(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}\right)p_0.$$

Standard Normal Distribution Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)



z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010