EGT2: IIA
ENGINEERING TRIPOS PART IIA

Friday 5 May 2017    9.30 to 11

**Module 3E3**

**RISK MODELLING**

*Answer not more than **two** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed.
Attachment: 3E3 Modelling Risk data sheet (3 pages).

**10 minutes reading time is allowed for this paper.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 (a) Students arrive at the only ATM machine available on a college campus at an average rate of 24 students per hour. The average service rate is 30 students per hour. Student arrivals and ATM service times are modelled using the Poisson distribution. Students wait in line and use the ATM according to first-come-first-served (FCFS).

(i) If a student starts using the ATM at 2:05 pm, what is the expected service completion time for the student? [5%]

(ii) A student arrives at 4 pm. What is the expected arrival time of the next student? [5%]

(iii) What percentage of time is the ATM busy? [5%]

(iv) What is the average number of students waiting in line? [5%]

(v) What are the chances that someone arriving at the ATM will have to wait in the queue? [5%]

(vi) If the opportunity cost of waiting is £10/hour per customer, what is the total hourly cost of customers' delays in queues to use the ATM? [10%]

(vii) What will be the average utilisation rate of the machines if a second ATM is installed next to the first one? [5%]

(viii) Suppose a second ATM is installed next to the first one. What is the probability that a student will not wait when he/she arrives? [10%]

(ix) Discuss how the average waiting time in the system changes as the number of servers increases in a system where everyone waits in the same queue (i.e. only one queue leads to all the servers). (Do not compute the average wait times.) [10%]

(b) To decide whether a company is discriminating against women, the following data were collected from the company's records: 'Salary' is the annual salary in thousands of pounds, 'Qualification' is an index of employee qualification, and 'Sex' is 1, if the employee is a man, and 0, if the employee is a woman.

Two linear models were fitted to the data and the regression outputs are shown below.

| **Model 1: Dependent variable is: Salary** | | | | |
|---|---|---|---|---|
| | Coefficients | standard error | $t$-stat | $P$-value |
| Intercept | 20,009.5 | 0.8244 | 24,271 | <0.0001 |
| Qualification | 0.935253 | 0.0500 | 18.7 | <0.0001 |
| Sex | 0.224337 | 0.4681 | 0.479 | 0.6329 |

| **Model 2: Dependent variable is: Qualification** | | | | |
|---|---|---|---|---|
| | Coefficients | standard error | $t$-stat | $P$-value |
| Intercept | -16,744.4 | 896.4 | -18.7 | <0.0001 |
| Sex | 0.850979 | 0.4349 | 1.96 | 0.0532 |
| Salary | 0.836991 | 0.0448 | 18.7 | <0.0001 |

Suppose that the usual regression assumptions hold.

(i) Are men paid more than equally qualified women? [10%]

(ii) Are men less qualified than equally paid women? [10%]

(iii) Do you detect any inconsistency in the above results? Explain. [10%]

(iv) Which model would you advocate if you were the defence lawyer? Explain. [10%]

2 (a) The classical knapsack problem of Operations Research is as follows: Given a set of items ($i$), each with an associated weight ($w_i$) and value ($v_i$), select the subset of items to maximise total value carried subject to the constraint that the total weight of items selected is less than or equal to some constant $W$. For example,

| Item ($i$) | Value ($v_i$) | Weight ($w_i$) |
|---|---|---|
| A | 70 | 40 |
| B | 75 | 30 |
| C | 40 | 35 |
| D | 50 | 25 |

$W = 80$

(i) Formulate this problem as a dynamic program by identifying the:

A. stage ($n = 1, \ldots, N$) and state ($s_n$); [5%]

B. decision ($x_n$) and constraint on $x_n$ as a function of $s_n$; [5%]

C. state transformation equation ($s_{n+1} = g(s_n, x_n)$) and the value of the initial state ($s_1$); [5%]

D. objective function over all $n$ stages ($\sum_n c(s_n, x_n)$) and the optimal recursion relationship [5%]

$$f_n^*(s_n) = \max\{c(s_n, x_n) + f_{n+1}^*(g(s_n, x_n))\}.$$

(ii) Solve this problem numerically, using dynamic programming. [40%]

(b) (i) Many analysts consider a scatter diagram to be an important first step in analysing an $X$-$Y$ relationship. Explain why this belief might be true. [10%]

(ii) "A high $r^2$ means a significant regression." Discuss this statement. [10%]

(iii) Discuss different measures of risk one can use for portfolio management. [10%]

(iv) Define and discuss the Markowitz portfolio analysis. [10%]

3 (a) I am trying to decide whether to move house or stay in my present one. I must decide immediately, but unfortunately there is an element of uncertainty in the situation, as my firm is in the middle of re-organisation, and may shortly move me to another site, closer to my present home than to the house I am considering buying. I reckon the chance of my being moved is about $p$ percent. If I am moved, I estimate that annual fares from my present home would cost £200. On the other hand, travel from my present home to the site I work at present costs me £300 per year. The cost to either site will be £240 a year from the new house. Assume that there is no price difference between the two houses, so that I make neither a profit nor a loss on the transaction.

(i) Build a decision tree for my decision. [10%]

(ii) For which values of $p$ should I stay in my current home? [10%]

(iii) I can secure additional information from one of my bosses. The boss's assessment of 'favourable' and 'unfavourable' is described by the following conditional probabilities: $P(a_1|s_1) = 0.8$ and $P(a_2|s_2) = 0.9$, where $a_1$ and $a_2$ represent the 'favourable' (the boss says we will move to a new site) and 'unfavourable' (the boss says we will not move to a new site) assessments, and $s_1$ and $s_2$ represent 'move to the new site' and 'do not move to the new site', respectively. Assuming the boss's assessment is favourable, for which values of $p$ would I stay in my current home? [15%]

(iv) Discuss the advantages and disadvantages of using decision analysis to solve my problem. [10%]

(b) At the beginning of each year, my car is in good, fair, or broken-down condition. A good car will be good at the beginning of the next year with probability 0.85 and fair with probability 0.10. A fair car will be a fair car at the beginning of the next year with probability 0.70 and broken down with probability 0.30. It costs £6,000 to purchase a good car; a fair car can be traded in for £2,000; and a broken-down car has no trade-in value and must immediately be replaced by a good car. It costs £1,000 per year to operate a good car and £1,500 to operate a fair car.

(i) Express the situation as a Markov Chain and specify the corresponding transition matrix whose states are the state of the car at the beginning of the year. [10%]

(ii) Define the concept of classes and find all classes for the Markov Chain. [10%]

(iii) Define the period for a state and derive the period for all states of the Markov Chain. [10%]

(iv) Determine the steady-state probability distribution. [10%]

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(v) Should I replace my car as soon as it becomes a fair car, or should I drive my car until it breaks down? Assume that the cost of operating a car during a year depends on the type of car on hand at the beginning of the year (after a new car, if any, arrives). [15%]

**END OF PAPER**