EGT2
ENGINEERING TRIPOS PART IIA

Tuesday 25 April 2017 9.30 to 11

Module 3F1

SIGNALS & SYSTEMS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider a discrete time system with input u and output y.

(a) (i) State sufficient conditions for the system to have a transfer function. What does it mean for the system to be *stable*? [10%]

(ii) A transfer function is said to be rational if it is of the form $p(z)/q(z)$ where p and q are polynomials in z. Suppose p can be written in the form

$$(a_0 + a_1 z^{-1} + ... + a_N z^{-N})q(z)$$

Is the system stable? Is the pulse response infinite or finite? Justify your answers briefly. [10%]

(b) Suppose a discrete time system is governed by the difference equation:

$$y_{k+1} + a y_k = u_k - b u_{k-1}$$

with $u_{-1} = 0, y_0 = \alpha$.

Using the z-transform, compute y_k for $u_k = \{1\}_{k \geq 0}$ (step input). [30%]

(c) (i) Let $\alpha = 0$ in the system in part (b). Compute the transfer function of the system. Under what conditions will the system be stable? [20%]

(ii) Keeping $\alpha = 0$, introduce feedback of the form $u_k = -c y_k + r_k$, where r_k is the new input to the system. Compute the transfer function of the new system. For the case $a = b$, provide conditions for stability in terms of a and c. Is the system causal? [30%]

2 The bilinear transform

$$s = \frac{z-1}{z+1}$$

is used to design digital filters $F_d(z)$ from analog prototypes $F_a(s)$.

(a) Suppose that $F_a(s)$ is a stable filter. Is $F_d(z)$ stable? Draw the region of the complex plane where the bilinear transform maps the stable poles of the analog prototype. Explain your answer with a mathematical argument. [25%]

(b) Explain the frequency warping $\omega = \tan(\Omega/2)$ between analog and digital domains. [20%]

(c) Use the bilinear transform to derive the transfer function of a highpass digital filter with normalised cutoff frequency 0.5π. Use the low-pass analogue prototype $\frac{1}{s+1}$ and the lowpass to highpass transformation $s = \frac{\omega_c}{\tilde{s}}$ with cutoff frequency at ω_c . Derive the expression of the difference equation that implements the filter. [30%]

(d) Find the normalized cutoff frequency to design the highpass filter with a cutoff frequency at 5Hz for a sampling period $T = 0.01$s. What is the normalised cutoff frequency if the sampling period is $T = 0.1$s? [10%]

(e) Discuss the phenomenon of aliasing. Find the maximum sampling period for real signals band-limited between 0Hz and 20Hz. [15%]

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3 Consider the finite impulse response filter $y_k = \gamma(u_k + u_{k-1} + u_{k-2})$ given by the sum of the last three values of the input multiplied by the gain γ. Assume γ is real.

(a) Write down the transfer function $G(z)$ relating the input $\{u_k\}$ and the output $\{y_k\}$. [10%]

(b) For $N = 4$, compute the expressions of the DFT samples

$$\bar{g}_p = \sum_{k=0}^{N-1} g_k e^{-j\frac{2\pi k}{N}p}$$

where $\{g_k\}$ is the impulse response of the filter $G(z)$. [15%]

(c) Take $\gamma = 1$. Determine which of the Bode diagrams in Fig. 2 corresponds to $G(z)$. Explain why you have excluded the other two diagrams. [15%]

(d) Using the Bode diagram you selected in (c), evaluate the steady state response of the filter as a function of γ, with input signals $u_k = 2$ and $u_k = \cos(2k)$.

[15%]

(e) One of the diagrams in Fig. 3 is the Nyquist diagram of $G(z)$ for $\gamma = 1$. Determine which one. Using Nyquist criterion, determine for what values of γ (if any) the closed loop in Fig. 1 is stable. [30%]

(f) Is the closed loop filter (input r, output y) in Fig. 1 a finite impulse response filter? (assume $\gamma = 1$). Explain your answer. [15%]

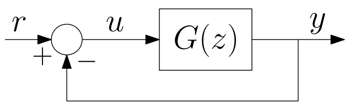

Fig. 1

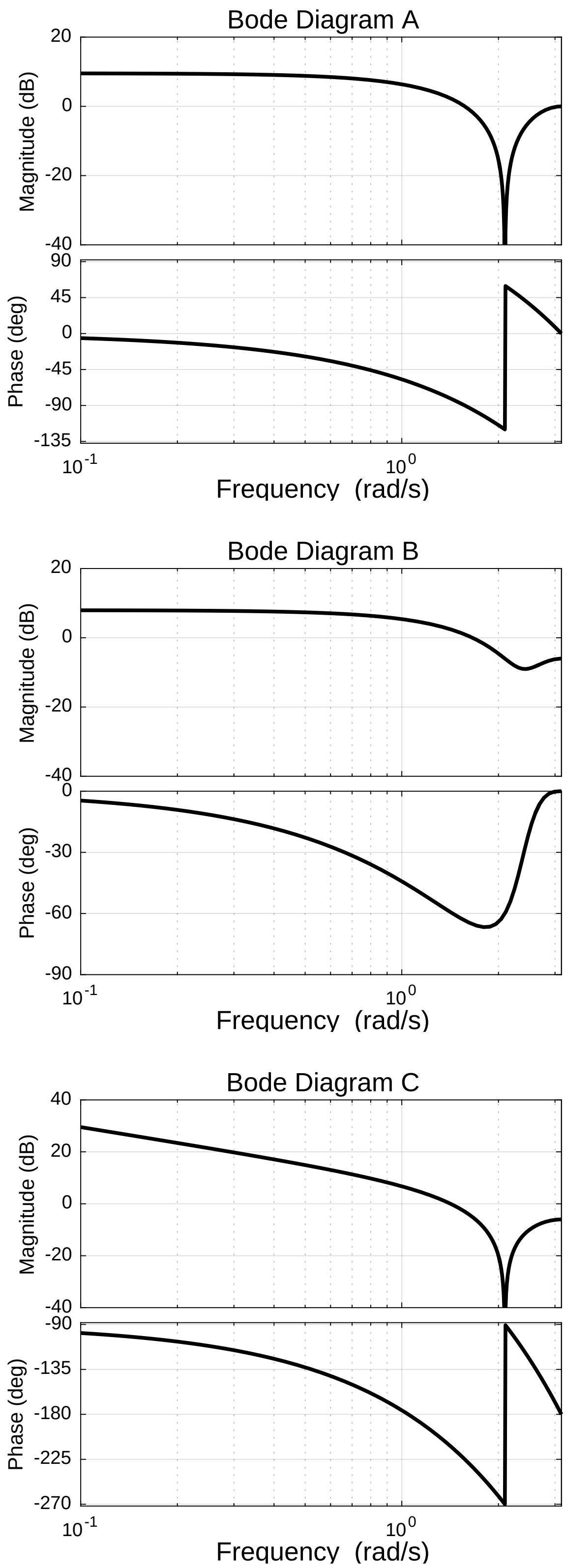
Bode Diagram A
Magnitude (dB)
20
0
-20
-40
Phase (deg)
90
45
0
-45
-90
-135
10-1
100
Frequency (rad/s)
Bode Diagram B
Magnitude (dB)
20
0
-20
-40
Phase (deg)
0
-30
-60
-90
10-1
100
Frequency (rad/s)
Bode Diagram C
Magnitude (dB)
40
20
0
-20
-40
Phase (deg)
-90
-135
-180
-225
-270
10-1
100
Frequency (rad/s)

Fig. 2

(cont.

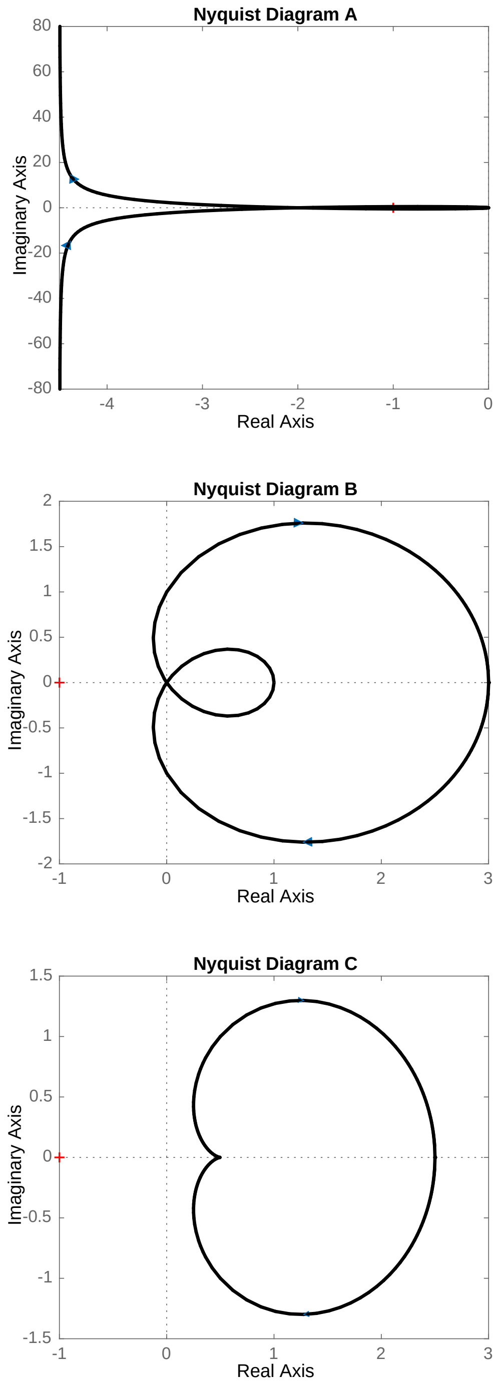
Nyquist Diagram A
Imaginary Axis
Real Axis
80
60
40
20
0
-20
-40
-60
-80
-4
-3
-2
-1
0
Nyquist Diagram B
Imaginary Axis
Real Axis
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
Nyquist Diagram C
Imaginary Axis
Real Axis
1.5
1
0.5
0
-0.5
-1
-1.5
-1
0
1
2
3

Fig. 3

4 (a) Let X be a continuous time random process.

(i) Define the autocorrelation function of X, r_{XX}. What does it mean for X to be *wide-sense stationary* (WSS)? [10%]

(ii) What does it mean for X to be *mean and correlation ergodic*? [10%]

(b) An RC circuit is subject to current fluctuations that are modelled as a white noise process, ε, with power spectral density (PSD) P_0. The voltage across the capacitor obeys

$$C\frac{dV}{dt}+\frac{V}{R}=\varepsilon$$

(i) What are the defining properties of a white noise process? [10%]

(ii) Calculate the PSD of V. [40%]

(iii) Compute the value of the autocorrelation of V at zero lag, $r_{VV}(0)$. [30%]

END OF PAPER

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