Version GV/4

EGT3
ENGINEERING TRIPOS PART IIA

Thursday 30 April 2015 2 to 3.30

Module 3F2

SYSTEMS AND CONTROL

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain the basis for controller design using observers and estimated state feedback as it applies to a state-space system of the form

$$\frac{d\underline{x}}{dt} = A\underline{x} + Bu$$
$$y = C\underline{x}$$

Mathematical results may be stated without proof. [30%]

(b) A force $u(t)$ is applied to a mass M, whose position $z(t)$ is measured. An observer is to be designed for its velocity.

(i) Show that the equations of motion can be written in the form

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \underline{x} + \begin{bmatrix} 1/M \\ 0 \end{bmatrix} u \qquad (1)$$
$$z = \begin{bmatrix} 0 & 1 \end{bmatrix} \underline{x} \qquad (2)$$

where the states are the position and velocity of the mass. [10%]

(ii) Denoting the state of the observer as $\hat{\underline{x}}$, write down the state-space equation of the observer in terms of its gain matrix H. [20%]

(iii) Find the transfer function of the observer, from u and z to $\hat{\underline{x}}_1$. If the relationship between z and u satisfies (1) and (2), under what further condition will the observer give an exact value of the velocity. [20%]

(iv) Now assume that the true mass of the system, M_{true}, is different to that assumed by the observer. How does the choice of observer gain matrix affect the accuracy of the estimate in this case? What other issues affect the choice of H? [20%]

2 (a) Discuss the reasons for using linearised models for control system design. When is the use of such models justified? [30%]

(b) Consider the state-space system

$$\dot{x}_1(t) = -x_2(t) + u(t)$$
$$\dot{x}_2(t) = x_1(t)^2 - 1$$

(i) Find all equilibrium points of this system. [10%]

(ii) Linearise the system about a general equilibrium x_{1e}, x_{2e}, u_e. [20%]

(iii) Sketch the state plane trajectories of the system when $u = 0$, paying particular attention to the regions in the vicinity of the equilibrium points. [40%]

3 (a) Consider the circuit of Fig 1, in which the operational amplifiers can be assumed to be ideal. Taking the voltages across the capacitors to be the states, verify that the state equation can be written as follows.

$$\frac{d\underline{x}}{dt} = \begin{bmatrix} -1/\tau_1 & 0 & 0 \\ 1/\tau_2 & 0 & 0 \\ -1/\tau_3 & -1/\tau_3 & -1/\tau_3 \end{bmatrix} \underline{x} + \begin{bmatrix} 1/\tau_1 \\ 0 \\ 0 \end{bmatrix} u$$

where $\tau_1 = R_1 C_1$, $\tau_2 = R_2 C_2$, $\tau_3 = R_3 C_3$. [20%]

(b) Find the transfer function from $\bar{u}(s)$ to $\bar{y}(s)$ for this system. [20%]

(c) Find conditions on τ_1, τ_2 and τ_3 such that the system is not controllable. In these cases, deduce the set of states that can be reached from $\underline{x}(0) = \underline{0}$. [20%]

(d) Find conditions on τ_1, τ_2 and τ_3 such that the system is not observable. In these cases, characterize the unobservable subspace. [20%]

(e) Comment on the connection between the conditions in parts (c) and (d) and the transfer function found in part (b). [20%]

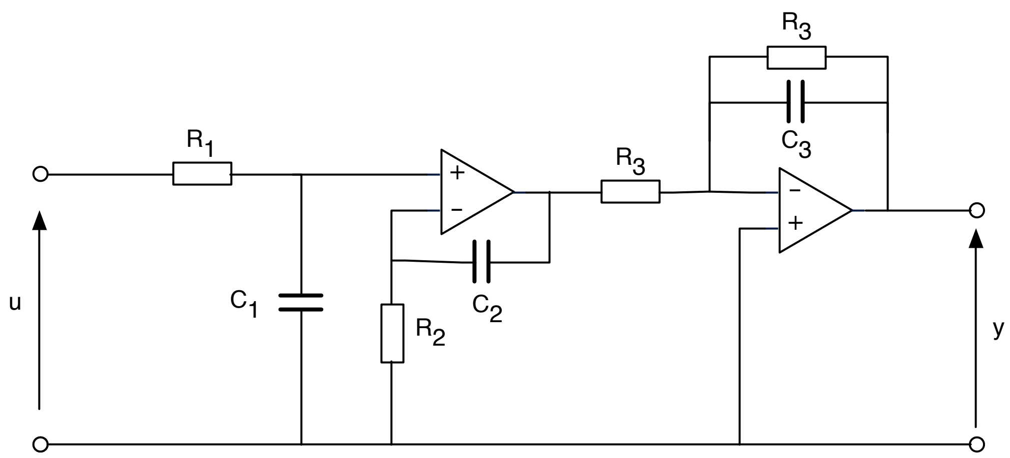

Fig. 1

4 Figure 2 shows the block diagram of a position control system, with reference $r(t)$ and measured position $y(t)$.

(a) Sketch one or more root-locus diagrams for this system, showing how the closed-loop poles vary with the gain K. You may assume that $0.025 < \alpha < 0.25$, but may need to treat different parts of this range separately. You should find expressions for the asymptotes and for the break-away points on the real axis. [35%]

(b) Explain the significance of α, and how it affects the achievable closed loop behaviour. [15%]

(c) For $\alpha = 0.25$, find the value of K such that the closed-loop poles have a damping ratio of $1/\sqrt{2}$. [25%]

(d) Fixing K at the value found in part (c), sketch a root locus diagram indicating how the closed loop poles move for values of α close to 0.25. [Hint: You might find it easier to write $\alpha = 0.25 + \delta$ and consider positive and negative δ.] [25%]

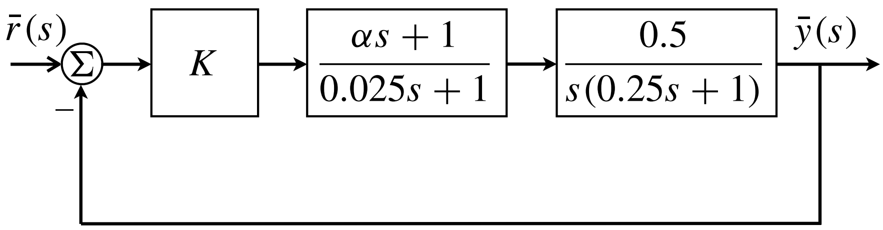

Fig. 2

END OF PAPER

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