EGT2
ENGINEERING TRIPOS PART IIA

Monday 2 May 2016   2 to 3.30

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Consider a platoon of n vehicles, where the speed of the ith vehicle is v_i as depicted in Fig. 1. The acceleration of the first (lead) vehicle is given by $\dot{v}_1 = u(t)$, where $u(t)$ is a control input, and the acceleration of subsequent (following) vehicles is given by

$$\dot{v}_i = k(v_{i-1} - v_i), \quad i = 2, \ldots, n.$$

The gain $k > 0$ is the same for all vehicles.

Fig. 1

(i) Consider a platoon of length $n = 3$. Write down a state-space model of the system, and determine if the system is observable from measurements of v_3 alone. [30%]

(ii) Repeat for $n = 4$. What can you infer about the observability of the system from the velocity of the final vehicle v_n for platoons of arbitrary length n? [20%]

(b) Let $k = 2$. Design a *state-space observer* to estimate the states of a platoon of length $n = 3$ from a measurement of the final vehicle's velocity, giving your answer as the individual equations for the rate of the change of the estimates of each velocity $\hat{v}_i$ (i.e. $\dot{\hat{v}}_1 = \ldots$ etc). The poles of the observer should all be placed at $s = -1$. [40%]

(c) For $k = 2$, and the observer of the previous section, a controller $u = k(V - \hat{v}_1)$ is now implemented. Here, V is a constant desired speed. Write down the location of all the closed-loop poles. [10%]

2 (a) (i) Define the notion of *controllability* for a state-space model. [10%]

(ii) Give two ways by which the controllability of a state-space model may be determined. [10%]

(b) Consider the system of three tanks shown in Fig. 2, where each tank has a unit cross-sectional area. Water is added to the second tank at volumetric rate u.

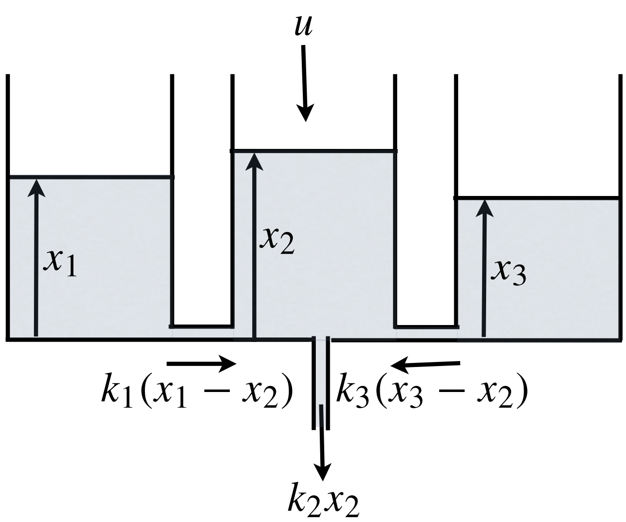

Fig. 2

(i) Show that the state space equations for this system can be written as

$$\dot{x} = \begin{bmatrix} -k_1 & k_1 & 0 \\ k_1 & -(k_1 + k_2 + k_3) & k_3 \\ 0 & k_3 & -k_3 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

[10%]

(ii) Under what conditions is the system controllable? [30%]

(iii) If $k_1 = k_3 = 1$ and $k_2 = 0$ then what is the subsequent response to the input $u(t) = \delta(t)$? [30%]

What is the set of reachable states for these parameters? [5%]

Comment on this result. [5%]

3 Figure 3 shows a pendulum free to rotate in a vertical plane. The pendulum is able to make more than one complete rotation, and $\theta(t)$ denotes its total cumulative rotation since $t = 0$. The equation of motion, including a term to represent damping, can be written as

$$\ddot{\theta} + 2\dot{\theta} + 8\sin\theta = 0.$$

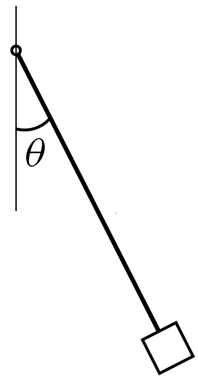

Fig. 3

(a) Write down a state space model of the system, with states $x_1 = \theta$ and $x_2 = \dot{\theta}$, and characterize all equilibria of the system. [20%]

(b) Linearize the system about each equilibrium, characterising each as stable or unstable. [50%]

(c) Hence, sketch the state plane trajectories of the system, paying particular attention to the regions in the vicinity of the equilibrium points. Your sketch should cover a range of x_1 from a little less than 0 to a little greater than 2π and show at least one trajectory that starts at a point with $x_1 = 0$, $x_2 > 0$ and finishes at a stable equilibrium with $x_1 \neq 0$. [30%]

4 A simplified model of the longitudinal dynamics of a proposed highly manoeuvrable aircraft is given by

$$\dot{x} = 2x + \dot{u}$$

where x is the vertical velocity of the aircraft and u is the elevator angle.

(a) Show that the transfer function from $\bar{u}(s)$ to $\bar{x}(s)$ is given by

$$G(s) = \frac{s}{s-2}.$$

The aircraft is to be stabilised using feedback, so that

$$\bar{u}(s) = -K(s)\bar{x}(s).$$

Explain, with reference to a root locus diagram, why any stabilising controller $K(s)$ must itself be unstable. What are the practical implications of this? [30%]

(b) Sketch the root-locus diagram for the feedback system when

$$K(s) = k\frac{s+2}{s-1}.$$

Find the range of values of k for which the feedback system is stable and the value of k for which the dynamics are critically damped. [40%]

(c) Suppose, instead, that $K(s)$ has the form

$$K(s) = k\frac{1}{(s+a)(s-1)}.$$

(i) Sketch the root locus diagram when $a = 1$. [20%]

(ii) Under what conditions on a is the feedback system stable for sufficiently large values of k? [10%]

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