Version GV/4

EGT2
ENGINEERING TRIPOS PART IIA

Friday 5 May 2017 2 to 3.30

Module 3F2

SYSTEMS AND CONTROL

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Define what it means for a system to be *controllable*. [20%]

(b) For a particular single central heating radiator, the outlet temperature T_{out} can be assumed to satisfy the differential equation

$$\frac{dT_{\text{out}}}{dt} = k_a(T_{\text{in}} - T_{\text{out}}) - k_b(T_{\text{in}} + T_{\text{out}})$$

where k_a and k_b are constants, T_{in} is the inlet temperature and the temperatures are measured with respect to some fixed reference.

Three identical such radiators are placed in series, with the inlet temperature of the second being equal to the outlet temperature of the first, and the inlet temperature of the third being equal to the outlet temperature of the second.

(i) Find a state-space description of the system of three radiators, with the states being the outlet temperatures of the three radiators and the input being the inlet temperature of the first radiator. [20%]

(ii) Find the set of steady-state outlet temperatures which can be achieved. [20%]

(iii) Is the system controllable from the inlet temperature of the first radiator? [20%]

(iv) Explain how an input $T_{\text{in}}(t)$ can be constructed to take the system from any initial condition to any of the states found in part (ii) at a specified time T, and hold it there. (You are not required to compute a closed-form expression for this input.) Are there any limits to how short the time T can be? Consider the possibility of both theoretical and practical limitations. [20%]

[Recall that the controllability grammian is defined as $W(t_1) = \int_0^{t_1} e^{A\tau} B B^T e^{A^T \tau}\, d\tau$]

2 (a) A system with transfer function

$$G(s) = \frac{s^2+1}{s^2+2}$$

is to be controlled with a constant gain feedback controller

$$K(s) = k$$

Paying careful consideration to the angle condition, sketch the root-locus diagram for the feedback system as k varies. Hence, or otherwise, deduce that there is no value of k for which the feedback system is asymptotically stable. [30%]

(b) An integral controller,

$$K(s) = \frac{k}{s}$$

is now used with the same $G(s)$. Sketch the root-locus diagram for this combination and deduce the range of k for which the feedback is system is stable. [40%]

(c) In an attempt to improve the damping of the feedback system, a controller of the form $K(s) = \frac{k(s+\alpha)}{s(s+1)}$ is now used instead. Using the angle condition again, determine the value of α that results in the feedback system having a closed-loop pole at $s = -1 - j$ for some value of k. [30 %]

3 A small permanent magnet of mass m is suspended in a tube above an electromagnet of length L as in Fig. 1, so that the force (vertically upwards) on the permanent magnet is given by

$$F(t) = ki(t)\left(\frac{1}{x(t)^2} - \frac{1}{(x(t)+L)^2}\right)$$

where x is the height of the permanent magnet above the top of the electromagnet and i is the current in the electromagnet. The tube constrains the mass to move vertically.

(a) Write down a state-space model of this system, with input i. [25%]

(b) Linearise this system about an equilibrium with $x = X$. What value of i is required to maintain this equilibrium (you may take g to equal 10ms^{-2}). [25%]

(c) If $m = 0.1\text{kg}$, $L = 0.1\text{m}$, $X = 0.05\text{m}$ and $k = 0.1\text{N}/(\text{A}/\text{m}^2)$, then design a state feedback controller to place the poles of the linearised closed loop system at $-1 \pm j$. Write down the full form of this controller, expressing i in terms of x and $\frac{dx}{dt}$. [25%]

(d) Modify the controller of part (c) so that the equilibrium is now at $X = 0.06\text{m}$ but the state feedback gains are unchanged. Where are the closed-loop poles now? [25%]

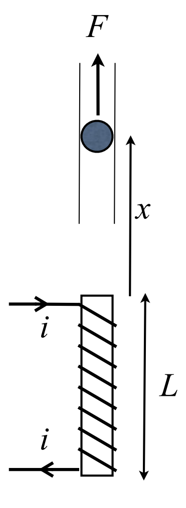

Fig. 1

4 Consider the feedback interconnection of the system $Y(s) = G(s)\big(U(s) + D(s)\big)$ with the controller $U(s) = K(s)Y(s)$. $G(s)$ has the state space realisation

$$\begin{aligned} \underline{\dot{x}} &= A\underline{x} + B(\underline{u} + \underline{d}) \\ \underline{y} &= C\underline{x} \end{aligned}$$

and $K(s)$ is an observer based controller with state feedback gain F and observer gain H. $D(s)$ represents an external disturbance signal.

(a) Write down the state space equations of the observer (with estimated state $\underline{\hat{x}}$) and the control signal and hence show that

$$K(s) = -F(sI - A + BF + HC)^{-1}H$$

[25%]

(b) Find the state space equations of the closed loop system, with state vector $\begin{bmatrix} \underline{x} \\ \underline{x} - \underline{\hat{x}} \end{bmatrix}$.

[25%]

(c) Where are the closed-loop poles, and under what conditions may they be freely assigned (give relevant tests)? [25%]

(d) Find the transfer function from $D(s)$ to $Y(s)$. Compare this to the case where full state feedback is available. [25%]

(Hint: Recall that $\begin{bmatrix} X & Z \\ 0 & Y \end{bmatrix}^{-1} = \begin{bmatrix} X^{-1} & -X^{-1}ZY^{-1} \\ 0 & Y^{-1} \end{bmatrix}$ when the indicated inverses exist.)

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