EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 27 April 2016   2 to 3.30

Module 3F4

DATA TRANSMISSION

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Draw a block diagram of a baseband digital transmission system showing transmit filter $H_T(\omega)$, receive filter $H_R(\omega)$ and channel response $H_C(\omega)$. Also indicate how the effect of channel noise may be included and state the usual assumptions made about the noise. [25%]

(b) The received pulse shape $p_R(t)$ satisfies the Nyquist pulse shaping criterion and the frequency response of the entire link is proportional to the specified pulse spectrum $P_R(\omega)$. Show that the expression for the energy of a transmitted pulse is:

$$E_T = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{k^2\,|P_R(\omega)|^2}{|H_C(\omega)|^2\,|H_R(\omega)|^2} d\omega$$

where k is a gain term.

Also show that the optimum receive filter is:

$$|H_R(\omega)| = \left| \frac{P_R(\omega)}{\sqrt{N(\omega)}H_C(\omega)} \right|^{\frac{1}{2}}$$

where $N(\omega)$ is the channel noise power spectral density. [50%]

(c) For the case where the noise power spectral density $N(\omega) = N_0$, the gain of the received pulse shape at the optimum sample instant (assumed at $t = 0$ without loss of generality) $p_R(0) = 1$, polar binary line coding is used with transmitted levels of $+1$ V and -1 V , and assuming equiprobable random binary data, determine an expression for the probability of bit error in terms of E_T and N_0 if:

$$|H_R(\omega)| = \sqrt{\frac{T}{\sqrt{N_0}}} \cos\left(\frac{\omega T}{4}\right) \qquad -\frac{2\pi}{T} \leq \omega \leq \frac{2\pi}{T}$$

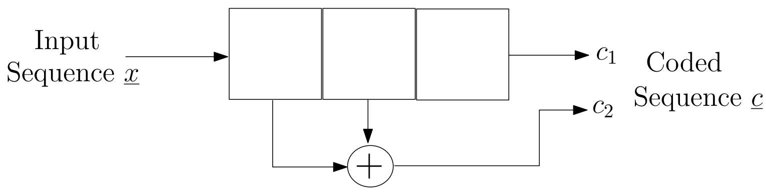
where T is the bit duration. [25%]

Note the Schwarz inequality:

$$\int_{-\infty}^{\infty} |F(\omega)|^2 d\omega \int_{-\infty}^{\infty} |G(\omega)|^2 d\omega \geq \left| \int_{-\infty}^{\infty} F(\omega)G(\omega)d\omega \right|^2$$

with equality when $F(\omega) = \lambda G^*(\omega)$ where λ is an arbitrary constant.

2 (a) A rate $\frac{1}{2}$ binary convolutional code is shown in the Figure below.



(i) Draw the first three stages of the trellis diagram of the code. [15%]

(ii) Highlight the path on the trellis for the input sequence 101, and find the corresponding coded sequence. [10%]

(b) A portion of the encoding table of a binary linear code is shown below.

$$100 \longrightarrow 10010$$
$$010 \longrightarrow 01011$$
$$101 \longrightarrow 10111$$

(i) Find the input blocklength, output blocklength, and the rate of the code. [10%]

(ii) Prove that the generator matrix of the code is

$$G = \begin{bmatrix} 1\,0\,0\,1\,0 \\ 0\,1\,0\,1\,1 \\ 0\,0\,1\,0\,1 \end{bmatrix}.$$

[25%]

(iii) Find a parity check matrix for the code. [10%]

(iv) What is the guaranteed error-correcting capability of the code? [10%]

(v) Construct a syndrome table and list the correctable error patterns. Comment on why this is quite a weak code. [10%]

(vi) Find an exact expression for the probability of decoding error when the code is used over a binary symmetric channel with crossover probability p. [10%]

3 (a) Explain why it is useful to represent a modulated real signal $s(t)$ in terms of a complex phasor signal $p(t)$, where

$$s(t) = \mathrm{Re}[\ p(t)\ e^{j\omega_C t}\ ]$$

where ω_C is the angular frequency of the carrier wave. [15%]

(b) Derive an expression for the spectrum, $S(\omega)$, of $s(t)$ in terms of the spectrum, $P(\omega)$, of $p(t)$. [15%]

(c) A phase-shift-keying (PSK) modulation scheme carrying random data has a symbol period of T_s and employs rectangular pulses of width T_s and amplitude a_0 as the signalling pulses $g(t)$. If the modulated pulses have zero mean and are uncorrelated with modulated pulses from other symbol periods, derive an expression for the power spectral density (PSD) of the resulting phasor waveform $p(t)$, and from this obtain the PSD of the resulting modulated signal waveform $s(t)$. [30%]

(d) Calculate the result of applying this expression to binary PSK (BPSK), quadrature PSK (QPSK), and 16-level PSK (16-PSK) and sketch the resulting power spectral densities of $s(t)$ for these three schemes, assuming a fixed data bit rate, R_b bit/s, and carrier frequency, $\omega_C/2\pi$ Hz, which is significantly greater than R_b. [20%]

(e) Discuss what important performance tradeoff is made when increasing the number of modulation states from 2 to 16, briefly indicating why this occurs. By means of constellation diagrams or otherwise, show how 16-level quadrature amplitude modulation (16-QAM) can give a better performance tradeoff than 16-PSK. [20%]

4 (a) Describe briefly how the phenomenon of multi-path radio transmission arises, and then explain what problems it causes for reception of high-speed binary data (such as for a digital radio or television signal) if conventional serial modulation techniques were to be employed. [20%]

(b) Why are multi-path delay spreads likely to be significantly greater at receivers that are located in moving vehicles than at receivers in fixed locations with directional antennae (e.g. in your home)? [10%]

(c) Explain with the aid of one or more block diagrams how orthogonal frequency-division multiplexing (OFDM) works, and how it is able to mitigate the problems of multi-path transmission. [20%]

(d) A digital audio broadcast (DAB) system is designed to transmit a composite audio data stream with a total bit rate of 1.2 Mbit/s. Error correction coding with a code rate of 1/2 is employed, together with OFDM which uses 1600 subcarriers, spaced 1 kHz apart and each employing quadrature phase-shift keying (QPSK) as the modulation method. Estimate the minimum symbol rate needed on each subcarrier to support the desired total coded bit rate, and hence determine the maximum multipath delay spread that can occur before intersymbol interference starts to degrade the performance of this system. You should briefly explain how guard periods are used to achieve this. [30%]

(e) Discuss how this tolerance to delay spread can permit single-frequency operation for a composite data stream of several national radio channels in a country with a radio network (like that in the UK) such that radio transmitters are typically spaced less than 80 km apart. What are the spectral efficiency advantages when such a single-frequency strategy is employed, compared to more conventional radio networks? [20%]

END OF PAPER

THIS PAGE IS BLANK