EGT2
ENGINEERING TRIPOS PART IIA

Friday 22 April 2016   2 to 3.30

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) In the context of medical ultrasound imaging, write short notes on each of the following:

(i) time-gain compensation; [15%]

(ii) image resolution and maximum scanning depth; [20%]

(iii) the material properties that cause ultrasound backscatter when they vary. [15%]

(b) The functions f and g are defined on the two-dimensional (x,y) plane as follows:

$$f(x,y) = \begin{cases} 2 & \text{for } 0 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$g(x,y) = \begin{cases} 1 & \text{for } 0 \le x < 2 \\ 0 & \text{otherwise} \end{cases}$$

(i) Calculate the Radon transforms of $f(x,y)$ and $g(x,y)$. [30%]

(ii) Comment on the similarity between your results in (i). Do these results indicate that it may be impossible to determine the width of stripe features in X-ray computed tomography images? Justify your answer. [20%]

2 (a) Figure 1 shows a laser range scanner, comprising a laser stripe source and a digital camera inclined at an angle α to the laser. The camera has focal length f and the focal point is an orthogonal distance L from the laser stripe. The laser stripe is reflected by an object and observed by the camera. x is the distance of each observation from the vertical centreline of the image. $L = 160$ mm, $f = 12$ mm, $\tan\alpha = 0.5$ and the pixel width (in the x direction) is $2/3$ mm.

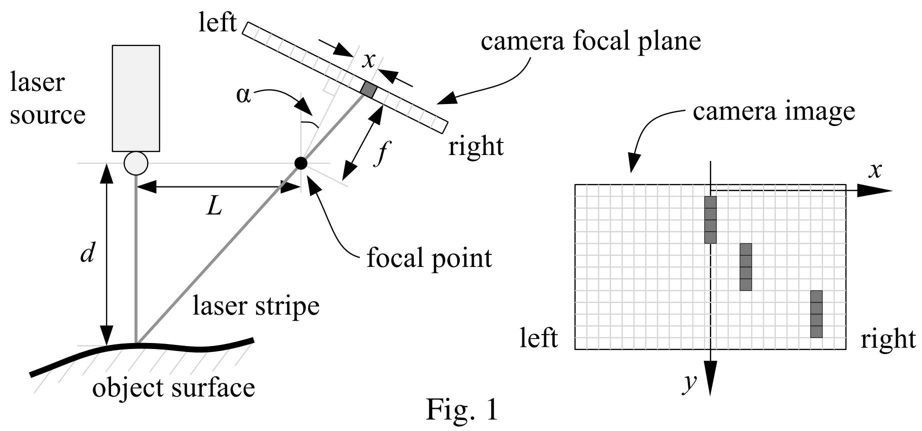

Fig. 1

(i) Show that the distance to the surface d is given by

$$d = \frac{L(f - x\tan\alpha)}{x + f\tan\alpha}$$ [20%]

(ii) The camera image in Fig. 1 shows reflections at twelve points, 0, 3 and 9 pixels from the centreline. Plot d against y for these points. [15%]

(iii) What are the minimum and maximum errors in d due to the discrete pixel width? [15%]

(b) $d(y)$ from (a)(ii) is approximated using a piecewise cubic B-spline, with basis

$$\mathbf{M} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

(i) Sketch this approximation on the plot you made in (a)(ii). [15%]

(ii) What is the maximum error in d introduced by this approximation? [25%]

(iii) Given your answers to (a)(iii) and (b)(ii), discuss the validity of using piecewise cubic B-splines for this application. [10%]

3 (a) In a computer graphics viewing system, the mapping from view coordinates to homogeneous 3D screen coordinates (i.e. the projection matrix) can be written as

$$\begin{bmatrix} wx_s \\ wy_s \\ wz_s \\ w \end{bmatrix} = \begin{bmatrix} d/x_{\max} & 0 & 0 & 0 \\ 0 & d/y_{\max} & 0 & 0 \\ 0 & 0 & -f/(f-n) & -fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$

Sketch the shape of the view volume in both coordinate systems. Annotate your sketch to show d, $y_{\max}$, n and f. [20%]

(b) Now consider the specific projection matrix

$$\begin{bmatrix} 3/4 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -100/99 & -1000/99 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

(i) Calculate the field of view, in degrees, in each of the x and y directions. [15%]

(ii) Calculate the distances to the near and far clipping planes. [10%]

(iii) Describe any geometrical distortion that may occur if the rendering is rasterized in a 640×480 pixel window. What about a 1920×1080 window? [15%]

(c) In a CAD package, the user clicks on a surface to select a particular triangle to edit. This is implemented by rendering the surface to an invisible, off-screen buffer using a narrow view volume that clips everything projecting more than one pixel, horizontally or vertically, away from the click point, and then noting which triangles survive.

(i) For the viewing example in (b), and for a click in the middle of a 640×480 window, calculate a suitable projection matrix for the off-screen rendering. [20%]

(ii) Describe, qualitatively, any difficulties you would have calculating the off-screen projection matrix had the click point not been at the centre of the window. [10%]

(iii) If more than one triangle survives the clipping operation, suggest a suitable strategy for selecting which triangle to edit. [10%]

4 (a) Explain, with the aid of diagrams, what is meant by *recursive ray tracing*. If there are n_r pixels, n_p polygons and n_l light sources, and d levels of inter-reflection are modelled, calculate the number of ray-polygon intersection tests required if, naively, every polygon is tested for intersection with every ray. [40%]

(b) One way to speed up ray tracing is to divide space into a regular voxel array and, for each voxel, pre-compute which polygons intersect that voxel. Then, for each ray, its path through the voxel array is established. Finally, for each voxel the ray encounters, only those polygons associated with that voxel are checked for intersection with the ray.

Consider the specific two-dimensional ray-tracing example in Fig. 2.

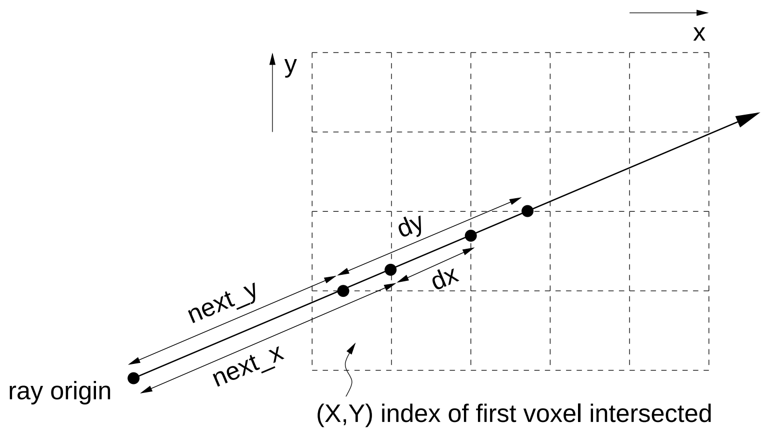

Fig. 2

(i) Write some pseudo-code to output an ordered list of the voxels the ray passes through. You may use as many of the quantities labelled in Fig. 2 as you like. You may assume that these quantities have been pre-computed. [20%]

(ii) Describe, qualitatively, how you would modify your code to work with rays at different angles. [10%]

(iii) Describe, qualitatively, how you would modify your code to work in three dimensions. [10%]

(c) By considering a ray-traced image of a teapot in an otherwise empty stadium, identify shortcomings of the approach in (b), and suggest how they might be remedied. [20%]

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Part IIA 2016

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) (i) $2 \operatorname{cosec} \theta$ for $\theta \neq 0$

3. (b) (i) $106.3°$

   (ii) $n = 10$, $f = 1000$

   (c) (i) $\begin{bmatrix} 240 & 0 & 0 & 0 \\ 0 & 240 & 0 & 0 \\ 0 & 0 & -100/99 & -1000/99 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

4. (a) $n_r n_p (1 + n_l)(2^{d+1} - 1)$