Version AHG/2

EGT2
ENGINEERING TRIPOS PART IIA

Wednesday 27 April 2022 9.30 to 11.10

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Consider the equation

$$\mu(x, y) = \int_0^{\pi} p_\phi(s) * q(s)\ d\phi$$

which is the essence of the filtered backprojection algorithm.

(i) Explain carefully what each term represents. [10%]

(ii) Show how the equation can be approximated as a discrete summation, and hence list the key steps involved in filtered backprojection. [20%]

(b) The equation in part (a) assumes parallel X-ray beams and is therefore not obviously applicable to the wide, fan-shaped beam utilized in modern CT scanners.

Figure 1(a) shows the starting position of a third generation scanner and the first set of projections recorded by the detectors. Note that the leftmost projection is vertical, there are n detectors and the fan beam angle is ϕ. Figure 1(b) shows the second set of projections, obtained by rotating the source and scanner through an angle $\Delta\theta$, while Fig. 1(c) shows the two sets of projections superimposed. If $\phi = (n-1)\Delta\theta$, then the second-from-left projection in (b) will also be vertical, as is evident in Fig. 1(c) and also in Fig. 1(d), which shows only the two X-ray source positions and the two vertical projections.

(i) How many rotations of $\Delta\theta$ are required to capture a complete set of vertical projections? [10%]

(ii) What is the width of the reconstructed field of view, i.e. the horizontal distance between the leftmost and rightmost vertical projections? Express your answer in terms of ϕ and R, the distance of the X-ray source from the centre of rotation. [10%]

(iii) How many rotations of $\Delta\theta$ are required to capture a complete set of parallel projections in all directions, for the purposes of backprojection? You may assume that $k\Delta\theta = \pi$ radians, for some integer k. [20%]

(iv) Given the set of parallel projections in (iii), is it possible to apply the filtered backprojection method in part (a)? Would any modifications be necessary? [15%]

(v) Explain why it is feasible to place a narrow, cylindrical collimator in front of each detector, and why this is desirable. [15%]

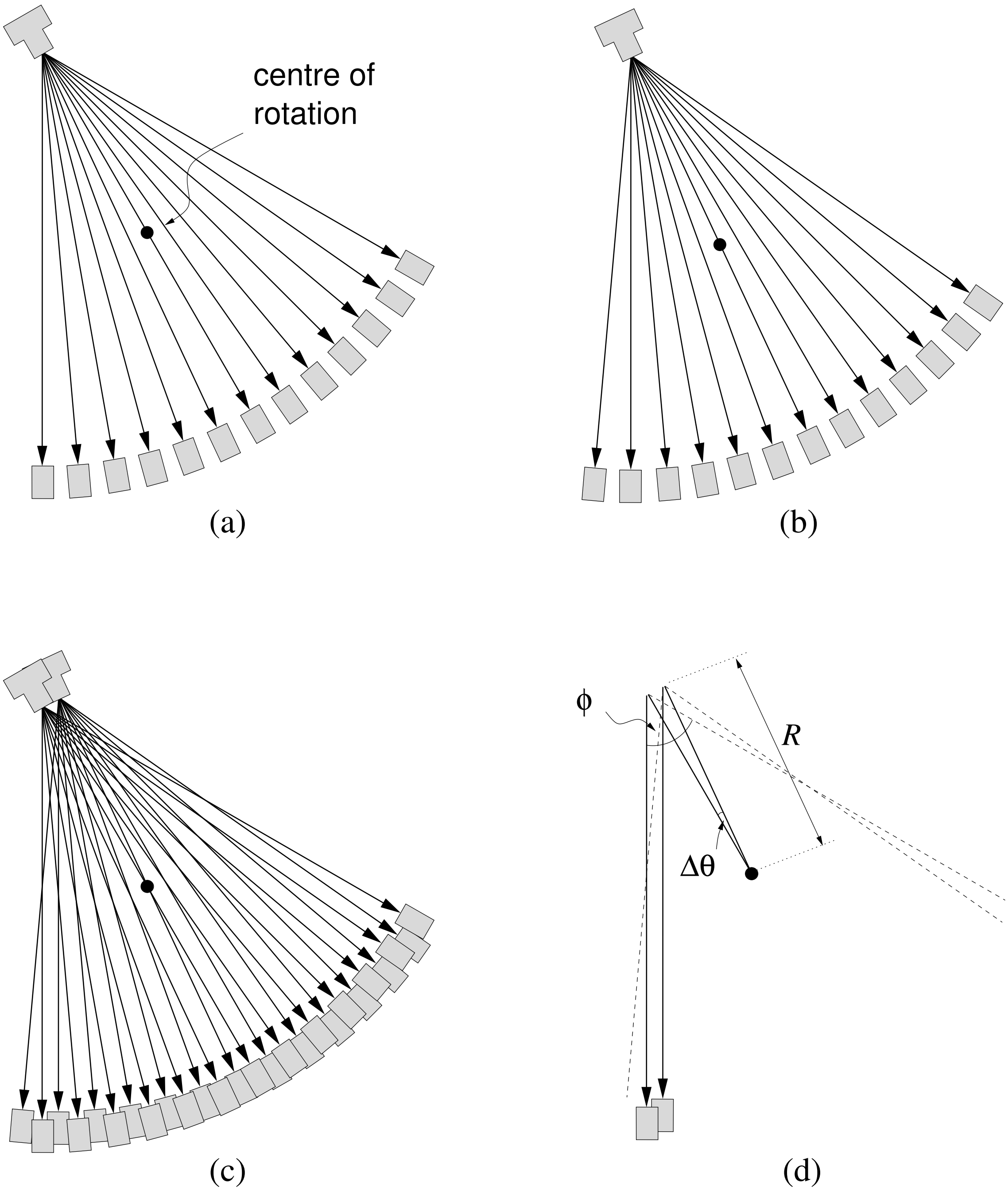

Fig. 1

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2 (a) Explain how the piecewise cubic parametric approach for curves can also be used to interpolate or approximate regularly spaced, one-dimensional scalar data. [20%]

(b) It is intended to use a piecewise cubic parametric function to resample scalar data with values $y = \{y_1,\ y_2,\ y_3,\ y_4\}$ at locations $x_y = \{-1,\ 0,\ 1,\ 2\}$, in order to generate new data $z = \{z_1,\ z_2,\ z_3\}$ at locations $x_z = \{0,\ 0.5,\ 1\}$.
The new data z can be represented as a weighted sum of the data, $\sum_{i=1}^{4} w_i y_i$, with a different set of weights w_i for each element $z_1 \ldots z_3$.
The basis matrices for the Catmull-Rom spline M_{CR} and the B-spline M_B are:

$$\mathrm{M}_{CR} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \quad \mathrm{M}_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

(i) For each element of z, determine the weights w_i if a Catmull-Rom spline is used to resample the data. [15%]

(ii) For each element of z, determine the weights w_i if a B-spline is used to resample the data. [15%]

(iii) What will the weights be if the data is linearly interpolated? What would be the equivalent basis matrix for linear interpolation within the piecewise cubic parametric framework? [25%]

(iv) Compare the weights in (iii) with those calculated in (i) and (ii). What does this reveal about the resampling results when using Catmull-Rom splines or B-splines? [15%]

(v) What is the significance of the convex hull property in this context, and how can you deduce whether this property holds by examining the weights w_i? [10%]

3 (a) Explain how the *Marching Cubes* algorithm can be used to extract a triangulated isosurface from a regular voxel array. [20%]

(b) Explain, with the aid of sketches, why some triangulation cases in Marching Cubes have alternative triangulations, and why the correct choice is important. [10%]

(c) Shape-based interpolation is sometimes applied to data before using Marching Cubes. How might this affect the resulting surface? [10%]

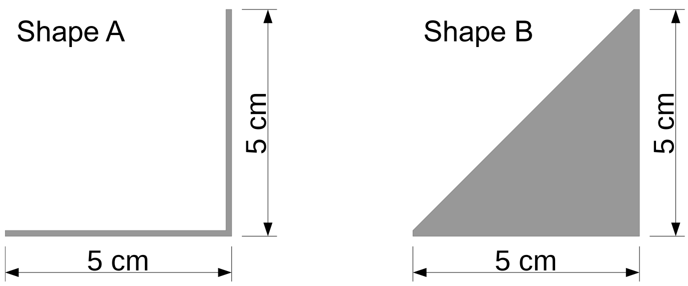

Fig. 2

(d) A distance transformation is to be applied separately to shape A and to shape B in Fig. 2 above.

(i) Sketch carefully the result of applying a city-block transform to these shapes and then extracting the iso-contour at a city-block distance of −5 cm (i.e. 5 cm outside the shape). [20%]

(ii) Repeat (i) using true Euclidean distances and extracting the contour at a Euclidean distance of −5 cm. Note where this agrees with the results in (i). [20%]

(iii) What is the maximum estimation error for city-block distances, and what would be the consequence of using city-block distance transforms when calculating a safety margin around a shape, for instance when planning radiotheraphy? [10%]

(iv) Explain to what extent this estimation error matters when using distance transformation in shape-based interpolation. [10%]

4 (a) In a computer graphics viewing system, the mapping from view coordinates to homogeneous 3D screen coordinates (i.e. the projection matrix) can be written as

$$\begin{bmatrix} wx_s \\ wy_s \\ wz_s \\ w \end{bmatrix} = \begin{bmatrix} d/x_{\max} & 0 & 0 & 0 \\ 0 & d/y_{\max} & 0 & 0 \\ 0 & 0 & -f/(f-n) & -fn/(f-n) \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$

Sketch the shape of the view volume in both coordinate systems. Annotate your sketch to show d, $y_{\max}$, n and f. [20%]

(b) Now consider the specific projection matrix

$$\begin{bmatrix} \tan(5\pi/12) & 0 & 0 & 0 \\ 0 & \tan(5\pi/12) & 0 & 0 \\ 0 & 0 & -1.5 & -3\times 10^5 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

in which distances are expressed in kilometres.

(i) Calculate the field of view, in degrees, in each of the x and y directions. [15%]

(ii) Calculate the distances to the near and far clipping planes. [10%]

(c) At dusk one day, the sun is setting on the horizon to the West (azimuth 270°, elevation 0°) while the moon is visible in the sky to the South-East (azimuth 135°, elevation 45°). An observer A is looking at the South-East horizon (azimuth 135°, elevation 0°) while a second observer B, at the same location, is looking directly at the moon. The moon is illuminated only by the sun. The distance between the observers and the moon is 4×10^5 km, and between the observers and the sun is 1.5×10^8 km.

(i) For observer A, find the view coordinates of the moon and the sun. Assume that the x_v axis is horizontal, the y_v axis is pointing vertically upwards, and the z_v axis is pointing backwards along the view plane normal. [15%]

(ii) By considering the 45° rotation around the x_v axis between A's and B's view coordinate systems, show that for observer B, the view coordinates of the moon and the sun are $(0\,,\,0\,,\,-4)\times 10^5$ km and $(1.5/\sqrt{2}\,,\,0.75\,,\,0.75)\times 10^8$ km respectively. [20%]

(iii) Observer B captures a perspective image of the moon using the projection matrix in (b). The resulting image is rasterized in a 500×500 pixel window and is illustrated in Fig. 3. By considering an arbitrary point P on the line joining the sun and the moon, find the angle θ to the horizontal at which the moon appears to be

 (cont.

illuminated. [Note that $\theta > 0$, so the moon appears to be illuminated from above, even though the sun is below the moon in the sky. This phenomenon is known as the *moon tilt illusion*.] [20%]

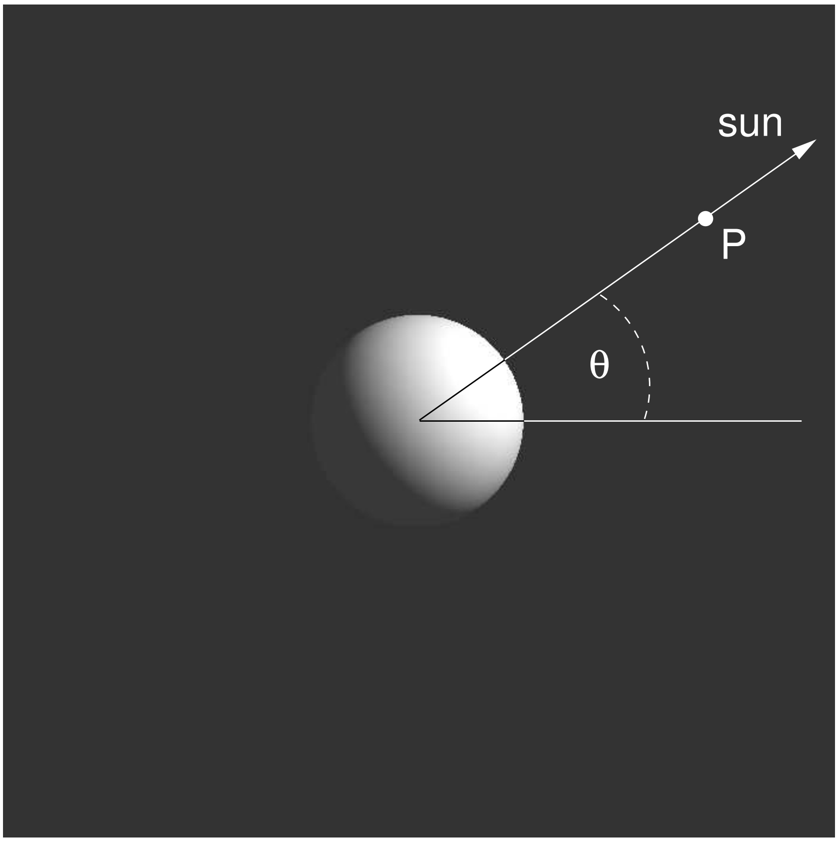

Fig. 3

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Part IIA 2022

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) (i) $n-1$ rotations of $\Delta\theta$
   (ii) $2R\sin(\phi/2)$
   (iii) $k+n-2$ rotations of $\Delta\theta$

2. (b) (i) $[\ 0\ 1\ 0\ 0\ ]^t$, $\frac{1}{16}[\ -1\ 9\ 9\ \ -1\ ]^t$ and $[\ 0\ 0\ 1\ 0\ ]^t$
   (ii) $\frac{1}{6}[\ 1\ 4\ 1\ 0\ ]^t$, $\frac{1}{48}[\ 1\ 23\ 23\ 1\ ]^t$ and $\frac{1}{6}[\ 0\ 1\ 4\ 1\ ]^t$
   (iii) weights $[\ 0\ 1\ 0\ 0\ ]^t$, $\frac{1}{2}[\ 0\ 1\ 1\ 0\ ]^t$ and $[\ 0\ 0\ 1\ 0\ ]^t$

   basis matrix $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

3. (d) (iii) $\sim 41\%$

4. (b) (i) $30°$
   (ii) $n = 2 \times 10^5$ km, $f = 6 \times 10^5$ km
   (c) (i) moon $(0\,,\ 4/\sqrt{2}\,,\ -4/\sqrt{2}) \times 10^5$ km, sun $(1.5/\sqrt{2}\,,\ 0\,,\ 1.5/\sqrt{2}) \times 10^8$ km
   (iii) $35.3°$