EGT2
ENGINEERING TRIPOS PART IIA

Thursday 25 April 2024    2 to 3.40

**Module 3G4**

**MEDICAL IMAGING & 3D COMPUTER GRAPHICS**

*Answer not more than **three** questions.*

*All questions carry the same number of marks.*

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Engineering Data Book

**10 minutes reading time is allowed for this paper at the start of the exam.**

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

**You may not remove any stationery from the Examination Room.**

1 (a) What do the acronyms SPECT and PET stand for? List the relative strengths and weaknesses of the two imaging modalities. [20%]

(b) For SPECT/PET, sketch approximate relationships between: (i) image signal-to-noise ratio (SNR) and measurement time; and (ii) image SNR and radionuclide dose. In each case, justify your answer. [20%]

(c) The equation below describes the process of attenuation correction in PET imaging.

$$N(d_1, d_2) = \exp\left[-\int_{d_1}^{d_2} \mu(v)\, dv\right] \int_{-\infty}^{+\infty} \lambda(s)\, ds$$

Describe the meanings of the various terms. How does the equation demonstrate that it is possible to use filtered backprojection to recover the spatial distribution of radioactivity within the patient? [20%]

(d) In a PET scanner, if two photons are detected within $\tau$ ns of each other, the event is recorded and used in the PET reconstruction. Typically, $\tau$ is of the order of 10 ns, even though modern detectors can time a scintillation to within around 0.5 ns. Discuss the advantages and disadvantages of reducing $\tau$ to substantially less than 10 ns. [20%]

(e) Figure 1 shows two adjacent detectors in a PET ring and the path of a 511 keV annihilation photon. Such energetic photons may pass through a substantial amount of crystal before being absorbed: in this example, the scintillation event happens in the second detector the photon encounters. What effect will this have on the resolution of the PET image? Will the effect be the same at all locations within the ring? [20%]

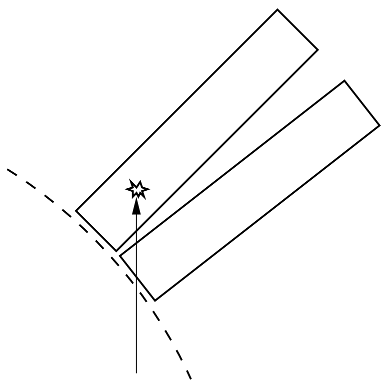

Fig. 1

2 (a) Define *interpolation* and *approximation*. Explain their respective advantages and disadvantages in the context of medical imaging data. [15%]

(b) Figure 2 shows four uniformly spaced samples, at locations $(s, t)$ with the given coordinates, whose scalar values are shown in bold. An interpolant is used to visualise the data between these samples along the dashed line $t = \frac{s}{2}$. In each of the following cases, sketch carefully, including axis labels, the interpolated value along the dashed line as $s$ varies from $0 \leq s \leq 1$, when:

(i) the data is interpolated using the nearest neighbour technique; [10%]

(ii) the data is interpolated using bilinear interpolation. [25%]

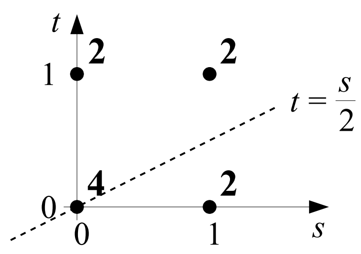


Fig. 2

(c) A Delaunay triangulation is used to create a simple structure from the four points in Fig. 2, in the region $0 \leq s \leq 1$, $0 \leq t \leq 1$, and the data is linearly interpolated within each triangle.

(i) Explain why there are two solutions for the Delaunay triangulation in this region, and sketch these solutions. [10%]

(ii) For each of the solutions in part (c) (i), sketch carefully, including axis labels, the interpolated value along the dashed line as $s$ varies from $0 \leq s \leq 1$. [20%]

(d) Suggest an alternative technique that could be used to *approximate* the data in Fig. 2. What are the consequences of having only four data values when approximating the data? [20%]

3 A cubic spline *curve* in one parameter $\mathbf{p}(t)$ is defined as $\mathbf{p}(t) = \mathbf{TMG}$.

(a) Define the terms in this equation, provide a similar equation which describes a spline *surface* $\mathbf{q}(s, t)$, and define any new terms in this new equation. [20%]

(b) A matrix $\mathbf{L}$ is given by

$$\mathbf{L} = \frac{1}{8}\begin{bmatrix} 8 & 0 & 0 & 0 \\ 4 & 4 & 0 & 0 \\ 2 & 4 & 2 & 0 \\ 1 & 3 & 3 & 1 \end{bmatrix}$$

Explain the process of *subdivision* and how the matrix $\mathbf{L}$ can be used to subdivide a Bézier surface patch. [20%]

(c) Two possibilities for the matrix $\mathbf{M}$ are:

$$\mathbf{M}_1 = \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad \mathbf{M}_2 = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

(i) For each of $\mathbf{M}_1$ and $\mathbf{M}_2$, demonstrate which of the rows of $\mathbf{G}$ determine the location and the gradient at the start of the curve $\mathbf{p}(t)$, i.e. at $t = 0$. [15%]

(ii) Explain how each of the elements of $\mathbf{G}$ define the curve when using each of $\mathbf{M}_1$ and $\mathbf{M}_2$, and hence what types of cubic spline $\mathbf{M}_1$ and $\mathbf{M}_2$ represent. [15%]

(iii) Where is the midpoint of the curve when using each of $\mathbf{M}_1$ and $\mathbf{M}_2$, in terms of the rows of $\mathbf{G}$ defined as $[P_1\ P_2\ P_3\ P_4]^T$? [10%]

(iv) Under what condition on $P_{1,\ldots,4}$ would the two midpoints calculated in part (c) (iii) be coincident, and where is this midpoint? [20%]

4 (a) The relationship between depth in 3D screen ($z_s$) and view ($z_v$) coordinates is

$$z_s = \frac{f(1 + n/z_v)}{f - n}$$

where $n$ and $f$ are the distances from the centre of projection to the near and far clipping planes respectively.

(i) Sketch a typical relationship between $z_v$ (on the $x$-axis) and $z_s$ (on the $y$-axis). [10%]

(ii) Why is this particular nonlinear relationship necessary for correct functioning of the surface rendering pipeline? [10%]

(iii) How do $n$ and $f$ affect the precision of the z-buffer algorithm? [10%]

(b) Figure 3 shows an OpenGL rendering of a thin, spherical shell, whose outer surface is dark grey and inner surface is light grey. The outer radius of the shell is 100 m and its thickness $t$ is 47 mm. The viewpoint is 1000 m from the centre of the shell, with the $z_v$ axis pointing directly away from the centre. The field of view is 30° horizontally and vertically, $n = 1$ m and $f = 1000$ m. A hardware $k$-bit integer depth buffer is used, with 0 representing $z_s = 0$ and $2^k - 1$ representing $z_s = 1$. There are rendering artefacts, with the light inner surface showing through the dark outer surface in places.

(i) Explain the cause of the artefacts, and why they do not appear to affect the visible outer edge of the shell. [15%]

(ii) Assuming that 47 mm is the largest value of $t$ for which the artefacts are evident at the centre of the rendering, estimate, in bits, the precision of the z-buffer. [25%]

(c) Discuss whether precision might be improved by using the $k$ bits of the depth buffer to store normalized floating point, rather than integer, representations of $z_s$, with some of the $k$ bits reserved for the mantissa and the remainder for the exponent. [30%]

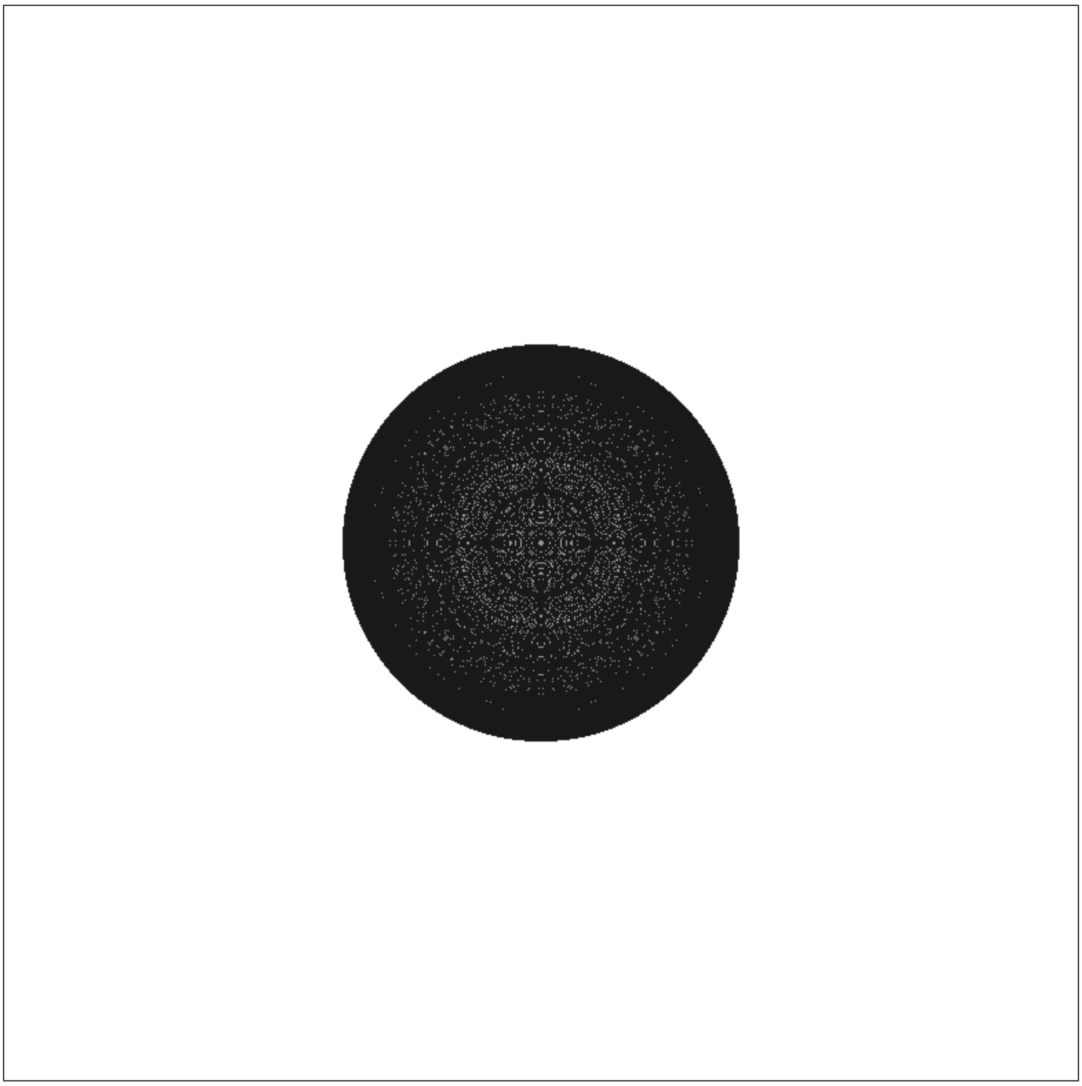

Fig. 3

**END OF PAPER**

**Part IIA 2024**

**Module 3G4: Medical Imaging & 3D Computer Graphics**

**Numerical Answers**

3. (c) (iii) $\frac{1}{8}(P_1 + 3P_2 + 3P_3 + P_4)$ , $\frac{1}{16}(-P_1 + 9P_2 + 9P_3 - P_4)$

(iv) $\frac{1}{2}(P_1 + P_4) = \frac{1}{2}(P_2 + P_3)$

4. (b) (ii) 24 bits