Version AHG/2

EGT2
ENGINEERING TRIPOS PART IIA

Thursday 1 May 2025   2 to 3.40

Module 3G4

MEDICAL IMAGING & 3D COMPUTER GRAPHICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book
Supplementary page with partially complete diagrams for Question 2

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) In the context of parallel-beam, two-dimensional, X-ray computed tomography, explain the meanings of the terms *projection* and *sinogram*. [25%]

(b) Suppose the X-ray beam has finite width t, so that what the scanner measures is a convolution of the actual projection and a pulse of width t. Sketch the Fourier transform of a pulse of width t. Hence, suggest an appropriate value for the distance Δs between the individual X-ray detectors in the CT detector array. Assume a linear array of detectors or, equivalently, a single source-detector pair that is translated a distance Δs at each step. [25%]

(c) Figure 1 illustrates the first part of the *Direct Fourier Reconstruction* algorithm. Explain what is happening in each of the steps and describe the missing final steps that allow recovery of the attenuation distribution μ. [25%]

(d) Suppose that, in the 2D frequency domain of Fig. 1(d), we require the worst case azimuthal resolution to be approximately the same as the radial resolution. In other words, we require the distance Δ_2 between the "spoke" tips to be approximately the same as the distance Δ_1 between the samples on each "spoke". Under this assumption, derive an approximate expression for the required number of projections in terms of the number of X-ray detectors. [25%]

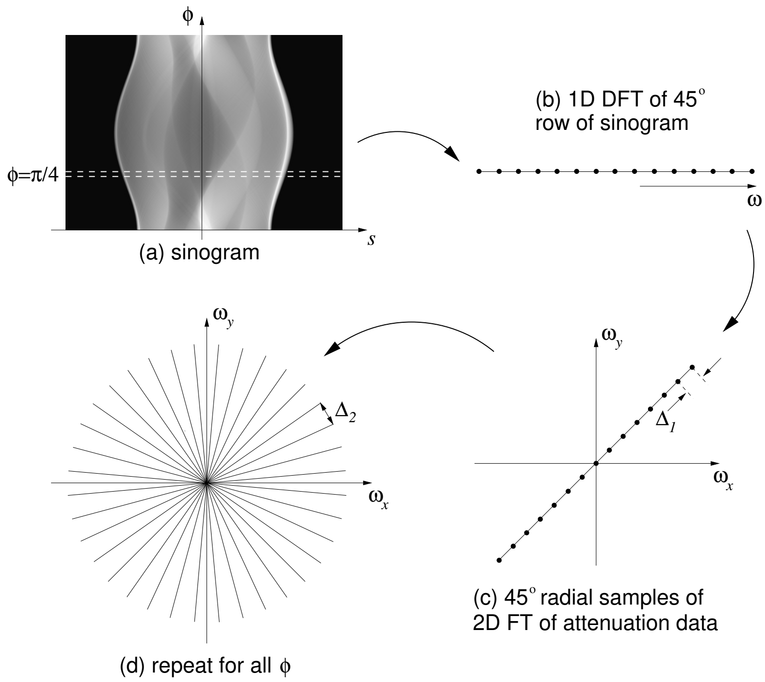


Fig. 1

2 Some bone is scanned, resulting in measurements of density sampled on a regular 2D grid, a section of which is shown in Fig. 2. The sampled density values are zero everywhere except for at two locations. Blank copies of this grid section are available on a separate sheet for the requested sketches.

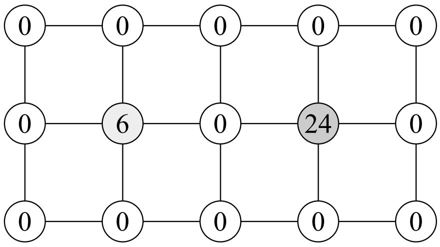


Fig. 2

(a) Marching Squares, using a threshold of three, is used to extract a polygonal representation of the underlying bone. Sketch the resulting bone contour. [15%]

(b) The data is smoothed to create new data at the same resolution (i.e. at the same sample locations). This is achieved by replacing each sample with a weighted sum of its neighbours. The weights **W** and the basis matrix for the B-spline **M** are:

$$\mathbf{W} = \frac{1}{36}\begin{bmatrix} 1 & 4 & 1 \\ 4 & 16 & 4 \\ 1 & 4 & 1 \end{bmatrix} \qquad \mathbf{M} = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}$$

(i) Show that using weights **W** represents B-spline approximation of the data. [20%]

(ii) Sketch the result of applying Marching Squares to this smoothed data using the same threshold of three as before. [20%]

(c) Instead of smoothing the data as in (b), it is interpolated to a much higher resolution grid, and then Marching Squares is applied to the higher resolution data (at the same threshold) to extract the bone contour. Sketch the resulting contours if using:

(i) nearest neighbour interpolation; [10%]

(ii) bilinear interpolation; [10%]

(iii) B-spline approximation. [10%]

(d) Which of the five approaches in (a) to (c) is likely to produce the most accurate representation of the bone and what do you consider to be wrong with the other approaches? [15%]

3 Figure 3 shows a diagram of a *freehand* laser scanner which will be used to scan the surface in Fig. 4. The plane of laser light is always directed at 90° to the line connecting the laser source and the focal point of the camera. This camera is mounted at a distance $L = 30$ cm and at an angle $\theta = 30°$ to the laser. It has focal length $f = 3$ cm and an image plane containing a sensor array of width $w = 2$ cm. The surface to be scanned is flat except for a cylindrical hole of diameter $t = 1$ cm and depth d.

(a) The plane of laser light is assumed to be very thin.

(i) What is the range of distances Z to the surface, measured from the laser source along the direction of light, over which it would ever be possible to see a reflection in the image plane? [25%]

(ii) What is the maximum depth d of the surface hole for which the scanner could measure the location of the base of the hole? Assume that the scanner is always in the optimal location and orientation for scanning each part of the surface. [20%]

(b) The plane of laser light is now assumed to have a width of 2 mm.

(i) What is the maximum error in Z over the flat part of the surface due to the laser thickness, and where would you see this error? Assume that the scanner is always in the optimal location and orientation for scanning each part of the surface. [20%]

(ii) Describe qualitatively, including diagrams as appropriate, how the laser thickness affects the maximum measurable depth d of the hole in the surface. [20%]

(c) What other sources of error are there in using this laser scanner and what can be done to reduce their impact? [15%]

(cont.

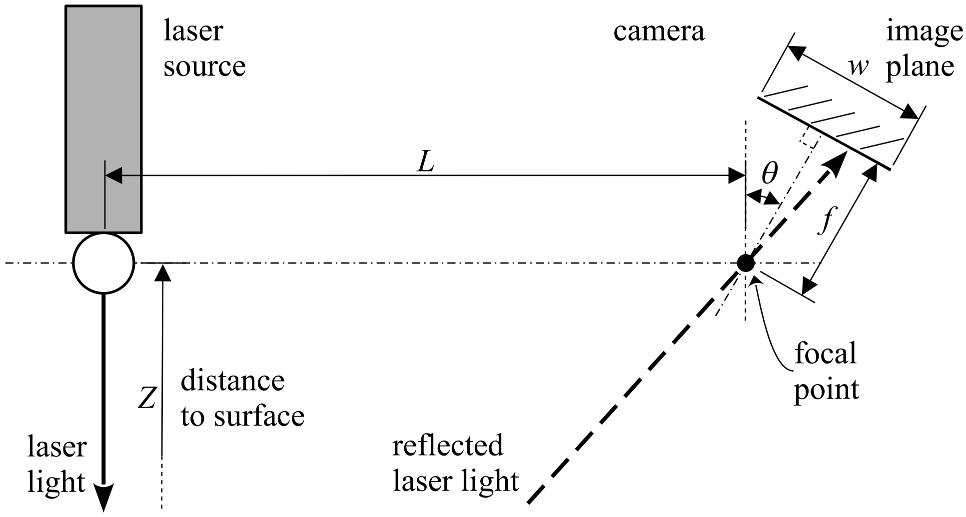


Fig. 3

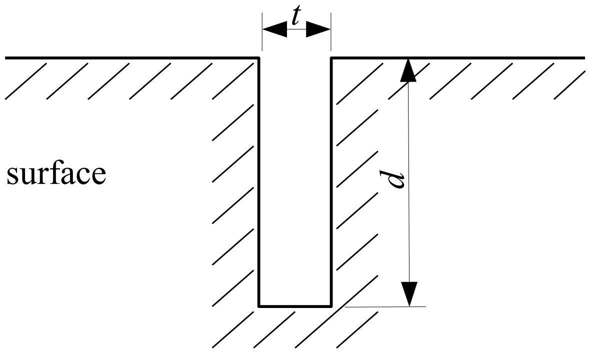


Fig. 4

(TURN OVER

4 (a) Explain briefly what is meant by *texture mapping* in computer graphics. Distinguish between 2D and 3D texture mapping and give one application of each variant. [25%]

(b) A triangle ABC is to be rendered with 2D texture mapping. The programmer associates vertex A with the texture coordinate $(0,0)$ and vertex B with $(600,0)$. A and B have 3D screen coordinates $(0,0.5,0.55)$ and $(0,0,0.99)$ respectively. The mapping from view coordinates to homogeneous 3D screen coordinates (i.e. the projection matrix) is given by

$$\begin{bmatrix} wx_s \\ wy_s \\ wz_s \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1.1 & -0.11 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_v \\ y_v \\ z_v \\ 1 \end{bmatrix}$$

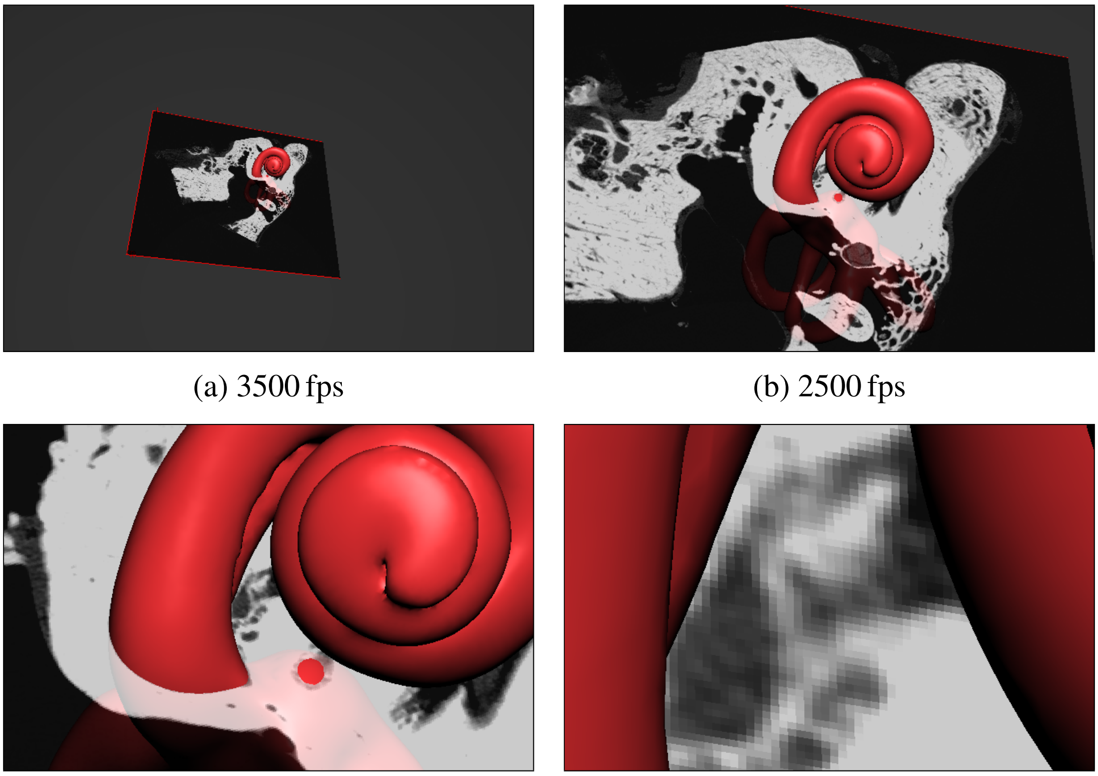
What texture coordinates should the rasterizer use for the point P, which lies exactly half way between A and B in the rendering? [40%]

(c) Repeat part (b) for the projection matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -0.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

How does this projection differ from the one in part (b)? [10%]

(d) Figure 5 shows four screenshots from a medical imaging visualisation and the corresponding rendering speeds expressed in frames per second (fps). In each view, there is an axial slice from the CT data set and a Phong-shaded surface rendering of some anatomical structures. The only difference between the four views is an isotropic scaling applied to all the objects in the scene. Discuss what might be responsible for the different frame rates and suggest how the rendering in Fig. 5(d) might be improved. [25%]



(a) 3500 fps (b) 2500 fps

(c) 2000 fps (d) 2500 fps

Fig. 5

END OF PAPER

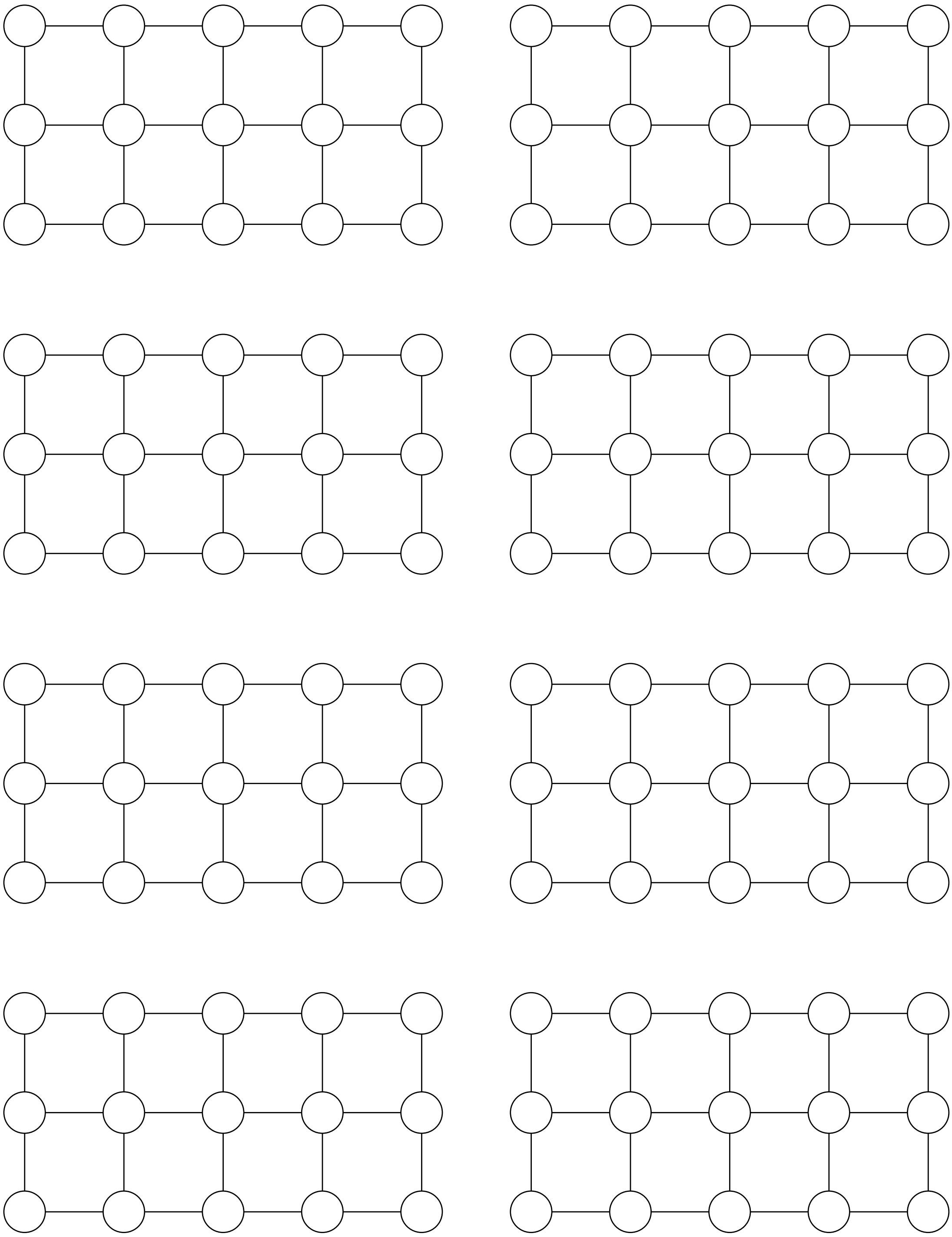
THIS PAGE IS BLANK

Version AHG/2 Candidate Number:

EGT2

ENGINEERING TRIPOS PART IIA

Thursday 1 May 2025, Module 3G4, Question 2.



Blank copies of Fig. 2 for Question 2.

These must be labelled with the appropriate question part and handed in with your answers.

Part IIA 2025

Module 3G4: Medical Imaging & 3D Computer Graphics

Numerical Answers

1. (b) $\Delta s \leq t/2$

   (d) $\sim \pi m/2$, where m is the number of X-ray detectors

3. (a) (i) 146.6 cm
   (ii) 4.94 cm

   (b) (i) 0.9 mm

4. (b) $(100, 0)$

   (c) $(300, 0)$