Version GRH/6

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 21 April 2015 9.30 to 11

Module 4A10

FLOW INSTABILITY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A10 Data Card (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 The characteristic vertical displacement, A_y , of a spring-supported, damped model of a structure, with characteristic vertical dimension D, in a horizontal flow of steady velocity U, is shown in Figure 1. The figure shows the response A_y/D plotted against the reduced velocity U/fD, where f denotes the frequency of vibration.

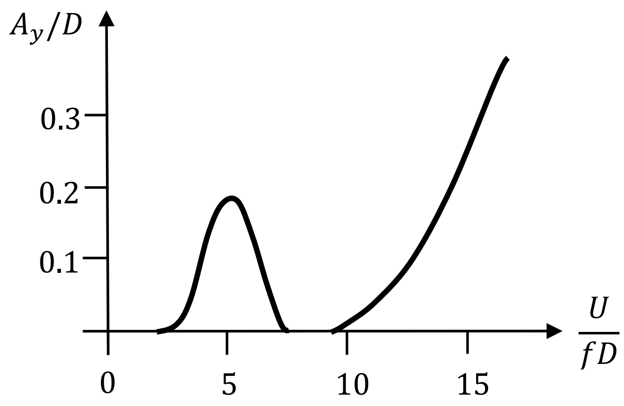

Figure 1. Sketch showing a typical response of a spring-supported damped structure.

(a) (i) The response shown in Figure 1 may be attributed to two distinct mechanisms of flow-structure interaction. What are the names of these mechanisms and what are the primary differences between them?

(ii) Redraw Figure 1 and annotate to show which mechanism causes which response. How could the response be damped in practice? Mark on your redrawn figure (as a dashed-line) the response you would expect if the damping of the system were increased.

[20%]

(b) Explain what the dimensionless quantity U/fD represents physically. Sketch the path traced out by the model for (i) $U/(fD) = 3$ and (ii) $U/(fD) = 6$. Annotate your sketches.

[20%]

(c) Explain what 'added mass' refers to in the context of flow-structure interaction. What is the link with the mass ratio? Comment on the significance of neglecting the added mass when examining the dynamics of a buoyant air-filled spherical shell (e.g. a ping-pong ball) that is free to rise in a stationary body of water.

[20%]

(d) Consider a long rigid horizontal cylinder (of diameter D) in horizontal steady flow of velocity U and density ρ. A classic spring-mass-damper system approach to predict the vertical displacement $y(t)$ of the cylinder (due to a lift force $F_L = \rho U^2 D C_L \sin(\omega_s t)/2$ which results from the shedding of vortices at a circular frequency ω_s , where $\omega_s = 2\pi f_s$), yields a dimensionless response of the form:

$$\frac{y}{D} = \frac{\rho U^2 C_L}{2k\sqrt{\left[1 - \left(\omega_s/\omega_y\right)^2\right]^2 + \left(2\zeta\, \omega_s/\omega_y\right)^2}} \sin(\omega_s t + \emptyset)$$

where the damping factor is ζ, the spring constant is denoted k , and ω_y denotes the natural circular frequency of vibration of the cylinder.

Show that the (dimensionless) resonant amplitude of vibration is

$$\frac{\rho U^2 C_L}{4k\zeta}$$

and deduce, with reasoning, why the amplitude at resonance may be regarded as independent of flow velocity and inversely proportional to reduced damping.

[40%]

2 A long slender cylindrical liquid thread, known as a capillary jet, is moving at a steady velocity in still air. The liquid has density ρ and the surface tension is γ.

Initially the cylinder has a radius $r = a$. When perturbed from this initial state, wave-like disturbances form, so that the radius of the thread varies as $r = \alpha + \beta\cos(kx)$, for some constant α. The coordinate x is measured along the axis of the jet, and β and k denote the amplitude and wavenumber of the disturbance, respectively, where $\lambda = 2\pi/k$ is the wavelength.

(a) For small amplitude disturbances, show that the ratio of the surface potential energy of the jet in the perturbed state to that in the initial state may be written as

$$\frac{PE_{perturbed}}{PE_{initial}} = 1 + \frac{\beta^2}{4a^2}(k^2a^2 - 1)$$

Hence, or otherwise, determine the wavelengths of disturbances that grow with time. [60%]

(b) Use dimensional arguments to establish an expression for the dimensionless growth rate of the disturbances as a function of the wavenumber k and the unperturbed radius a. Include brief explanations to clarify your workings.

[20%]

(c) Plotting dimensionless growth rate against dimensionless wavenumber, sketch the variation of the growth rate that is observed in practice. What is the physical significance of the maximum growth rate?

[20%]

3

(a) Consider the classic Taylor-Couette problem, namely, of a fluid in the narrow gap between two long concentric co-rotating circular cylinders. Provide a physical argument to establish that the flow is stable provided

$$\frac{d}{dr}(\Omega r^2)^2 \geq 0$$

throughout the fluid, where Ω denotes the angular velocity of the fluid. You may assume that the radial pressure gradient

$$\frac{\partial p}{\partial r} = \frac{u_\theta^2}{r}$$

where u_θ denotes the circumferential velocity.

[25%]

(b) Consider a two-dimensional inviscid mixing layer with a velocity profile

$$\boldsymbol{u} = \begin{cases} U_1 \boldsymbol{i} & z > 0 \\ U_2 \boldsymbol{i} & z < 0 \end{cases}$$

as shown in Figure 2.

Investigate the temporal stability of this flow by considering small disturbances to the interface of the vortex sheet of the form

$$\eta(x,t) = \hat{\eta} e^{ikx+st}$$

for wavenumber k and growth rate s , to show that the dispersion relationship is

$$s = -\frac{1}{2} ik(U_1 + U_2) \pm \frac{1}{2} k(U_1 - U_2)$$

[75%]

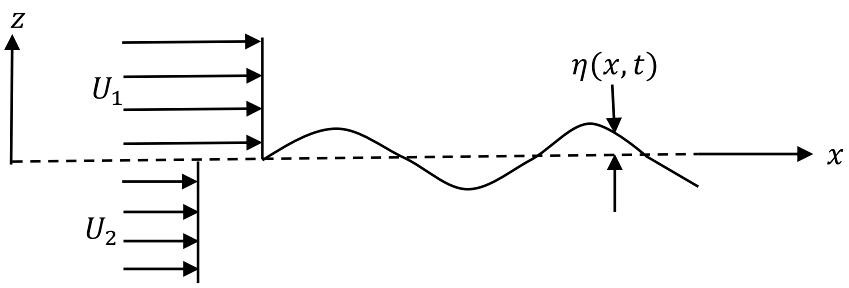

Figure 2. Schematic of velocity profile.

4 The one-degree-of-freedom galloping model shown in Figure 3, of transverse dimension D, is exposed to a horizontal flow of steady velocity U and density ρ. The equation governing the motion of the model as it responds to the net vertical aerodynamic force $F_y = \rho U^2 D C_y/2$ is

$$m\ddot{y} + \left(2m\zeta\omega_y\right)\dot{y} + ky = F_y \qquad (1)$$

In Equation 1, a dot signifies differentiation with respect to time and the displacement y is measured vertically downward, as indicated in Figure 3. The mass per unit length of the model (including the added mass) is denoted by m, ζ denotes the damping factor, ω_y the natural circular frequency of the cylinder and k the spring constant.

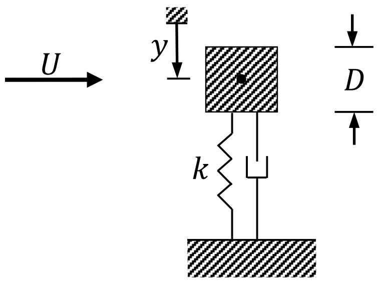

Figure 3. The spring-supported, damped, one-degree-of-freedom galloping model.

(a) Describe the mechanism that gives rise to the 'galloping' of a structure.

[15%]

(b) Explain what is meant by 'soft' and by 'hard' excitation. Give an example of a physical mechanism that could give rise to the hard excitation of a structure, e.g. the deck of a bridge.

[15%]

(c) Show that the vertical force coefficient C_y is related to the drag force coefficient C_D and the lift force coefficient C_L via

$$C_y = -\frac{U_{rel}^2}{U^2}\left(C_L \cos\alpha + C_D \sin\alpha\right)$$

 (cont.

where U_{rel} denotes the velocity of the fluid relative to the moving model.

[20%]

(d) Explain the physical significance of fluid forces that give rise to a damping term that is negative in the equation governing the motion of the model.

Show that for small angles of attack α , the model will be stable to galloping if

$$\frac{\partial C_y}{\partial \alpha} < 0$$

and deduce the conditions required for the onset of galloping.

[50%]

END OF PAPER

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