Version AA/5

EGT3
ENGINEERING TRIPOS PART IIB

Monday 23 April 2018 9.30 to 11.10

Module 4A15

AEROACOUSTICS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
Engineering Data Book
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (6 pages).

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An earthquake can result in surface waves of significant amplitude. These waves radiate sound into the atmosphere. As a result of the Sumatran earthquake in 2004, surface waves with a vertical velocity amplitude of $1\,\mathrm{cm\,s^{-1}}$ and a time period of $20\,\mathrm{s}$ were recorded. The speed of the surface wave was eight times the speed of sound in air. In the following, assume that the Earth is flat and the surface waves are plane and have a constant amplitude.

(a) Determine the direction of propagation of the plane sound wave. [20%]

(b) Determine the mean intensity of the plane waves radiated into the atmosphere. Assume, at sea level, the atmosphere has a mean density of $\rho_0 = 1.2\,\mathrm{kg\,m^{-3}}$ and speed of sound of $c_0 = 340\,\mathrm{m\,s^{-1}}$. [60%]

(c) Seismic activity can be detected in space. By applying conservation of energy of the sound wave and neglecting sound dissipation, refraction and reflection for these low-frequency waves, find the velocity amplitude induced by the sound wave in the ionosphere. Assume that the gas density in the ionosphere is $10^{-8}\rho_0$. Because the speed of sound does not vary as drastically as the density with altitude, assume that c_0 is a constant for the purpose of getting an order of magnitude estimate. You may also assume that the spatial extent of the surface wave is much longer than the distance from the surface of the Earth to the ionosphere, so that the waves remain plane with no spherical spreading. [20%]

2 Free reeds, found in musical instruments such as the harmonica, produce a tone when air is sucked through them because the flow makes the reed oscillate within the reed block at its resonant frequency. This modulates the opening of the reed resulting in a pulsating mass flow. In the following assume that the unsteady mass flow per unit volume at the reed is $\dot{\mu}(\mathbf{x},t)$, where $\mathbf{x}$ is the position and t is the time (a dot over a letter denotes a time derivative).

(a) Show that the equation governing sound propagation from the reed is given by

$$\frac{\partial^2 \rho'}{\partial t^2} - c_0^2 \nabla^2 \rho' = \ddot{\mu}(\mathbf{x},t)$$

where ρ' is the acoustic density perturbation and c_0 is the ambient speed of sound. [20%]

(b) If the unsteady mass flow through the reed is

$$\dot{m} = \int_V \dot{\mu}(\mathbf{y},t)\,d\mathbf{y}$$

where V is the source volume, then assuming that the reed is a spatially compact source radiating sound into free space, show that the acoustic pressure field radiated at a large distance r from the reed is given by

$$p'(r,t) = \frac{\ddot{m}(t - r/c_0)}{4\pi r}$$

[40%]

(c) Assuming $\dot{m}(t) = \rho_0 Q[1 + \cos(2\pi f t)]$, where $\rho_0 = 1.2\ \mathrm{kg\,m^{-3}}$ is the ambient density of air and $Q = 12$ L/min is the flowrate, $f = 261.6$ Hz is the resonant frequency of the reed, calculate the SPL of the sound radiated at a distance of 10 m from the reed. [30%]

(d) If, instead of sucking air through the reed, the reed is mechanically plucked it makes very little sound. Explain why this is the case. [10%]

3 Acoustic pressure fluctuations, p', in a fluid which has a uniform mean flow in the x_3-direction at a Mach number M satisfy the convective wave equation

$$\nabla^2 p' - \left(\frac{1}{c_0^2} \frac{\partial^2 p'}{\partial t^2} + \frac{2M}{c_0} \frac{\partial^2 p'}{\partial t \partial x_3} + M^2 \frac{\partial^2 p'}{\partial x_3^2} \right) = 0$$

where c_0 is the speed of sound.

A rigid walled duct of rectangular cross-section has a mean flow through it with Mach number M in the x_3-direction. The duct has width a and height b. The coordinate system is shown in Fig 1.

(a) Explain why the pressure perturbation in acoustic modes of frequency ω within the duct can be written in the form

$$p'(\mathbf{x},t) = e^{i\omega t} g(x_3) \cos\left(\frac{m\pi x_1}{a}\right) \cos\left(\frac{n\pi x_2}{b}\right)$$

for some function $g(x_3)$ and integer constants m and n. Detailed calculations are not required. [10%]

(b) By trying a solution of the form $g(x_3) = A\exp(ikx_3)$, where A and k are constants, find the function $g(x_3)$ in terms of $k_0 (= \omega/c_0)$ and M. [60%]

(c) Interpret your solution for $g(x_3)$ when $m = n = 0$. [15%]

(d) For a mode with non-zero m and/or n, determine the frequency range for which that mode is cut-off. [15%]

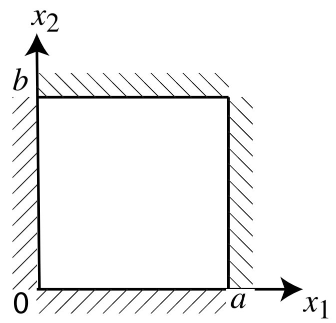

Fig. 1

4 Consider a wall-bounded two-layer fluid system, in which the region $x < 0$ contains a fluid of density ρ_0 and sound speed c_0, the region $0 < x < h$ contains a fluid of density ρ_1 and sound speed c_1, and an infinite wall with complex impedance Z is located at $x = h$. In $x < 0$ an incident plane sound wave of amplitude I and frequency ω propagates parallel to the x axis.

(a) Show that the reflected wave in $x < 0$ has complex reflection coefficient

$$R = I\frac{(1-Z^*)(1+\rho^*)+(Z^*+1)(1-\rho^*)\exp(2\mathrm{i}k_1 h)}{(1-Z^*)(\rho^*-1)-(Z^*+1)(1+\rho^*)\exp(2\mathrm{i}k_1 h)}$$

where

$$\rho^* = \frac{c_1\rho_1}{c_0\rho_0},\ Z^* = \frac{Z}{c_1\rho_1}$$

and $k_1 = \omega/c_1$. [80%]

(b) Calculate the modulus and phase of R/I when $Z^* = 1$. Comment on the physical implication of your results. [20%]

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Module 4A15 Aeroacoutics Data Sheet

USEFUL DATA AND DEFINITIONS

Physical Properties

Speed of sound in an ideal gas $\sqrt{\gamma RT}$, where T is temperature in Kelvins

Units of sound measurement

$$\begin{aligned}
\text{SPL (sound pressure level)} &= 20\log_{10}\left(\frac{p'_{rms}}{2\times10^{-5}\text{Nm}^{-2}}\right)\text{dB} \\
\text{IL (intensity level)} &= 10\log_{10}\left(\frac{\text{intensity}}{10^{-12}\text{watts m}^{-2}}\right)\text{dB} \\
\text{PWL (power level)} &= 10\log_{10}\left(\frac{\text{sound power output}}{10^{-12}\text{watts}}\right)\text{dB}
\end{aligned}$$

Definitions

Surface impedance Z_s, relates the pressure perturbation applied to a surface, p', to its normal velocity v'; $p' = Z_s v'$

Characteristic impedance of a fluid $\rho_0 c_0$

Specific impedance of a surface $Z_s/(\rho_0 c_0)$

Wavenumber $k = \omega/c_0 = 2\pi/\lambda$, where λ is the wavelength

Helmholtz number (or compactness ratio) = kD, where D is a typical dimension of the source.

Strouhal number $= \omega D/(2\pi U)$ for sound of frequency ω (in rad/s), produced in a flow with speed U, length scale D.

Basic equations for linear acoustics

Conservation of mass

$$\frac{\partial \rho'}{\partial t} + \rho_0 \nabla \cdot \mathbf{v}' = 0$$

Conservation of momentum

$$\rho_0 \frac{\partial \mathbf{v}'}{\partial t} + \nabla p' = 0$$

Isentropic

$$c_0^2 = \left.\frac{dp}{d\rho}\right|_S$$

Wave equation

$$\frac{1}{c_0^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = 0$$

Energy density

$$e = \frac{1}{2}\rho_0 v'^2 + \frac{1}{2\rho_0 c_0^2} p'^2$$

Intensity $\mathbf{I} = p'\mathbf{v}'$

Velocity potential ϕ' satisfies the wave equation and $p' = -\rho_0 \frac{\partial \phi'}{\partial t}$, $\mathbf{v}' = \nabla \phi'$.

Autocorrelation $F(\xi)$, the autocorrelation of $f(y)$ is given by

$$F(\xi) = \overline{f(y)f(y+\xi)}$$

$$F(0) = \overline{f^2}$$

Integral length scale, l

$$l\overline{f^2} = \int_{-\infty}^{\infty} F(\xi)d\xi$$

Sound power

Sound power from a source is defined as

$$P = \int_S \bar{\mathbf{I}} \cdot \mathbf{dS} = \int_{S_\infty} \frac{\overline{p'^2}}{\rho_0 c_0}\mathbf{dS}$$

for a statistically stationary source. For an outward propagating spherically symmetrical sound field $P = \frac{\overline{p'^2}}{\rho_0 c_0} 4\pi r^2$, where p' is the acosutic pressure at radius r.

For a sound field, which is a function of spherical polar coordinates r, θ only, and is independent of the azimuthal angle,

$$P = 2\pi r^2 \int_0^{\pi} \frac{\overline{p'^2}}{\rho_0 c_0} \sin\theta d\theta$$

Simple wave fields

1D or plane wave

The general solution of the 1D wave equation is $p'(x,t) = f(t - x/c_0) + g(t + x/c_0)$, where f and g are arbitrary functions. In a plane wave propagating to the right $p' = \rho_0 c_0 u'$; in a plane wave propagating to the left $p' = -\rho_0 c_0 u'$, u' being the particle velocity.

Spherically symmetric sound fields

The general spherically symmetric solution of the 3D wave equation is

$$\phi'(r,t) = \frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r},$$

where r is the distance from the source; f and g are arbitrary functions.

$\cos\theta$ dependence

The general solution of the 3D wave equation with $\cos\theta$ dependence is

$$p'(\mathbf{x},t) = \frac{\partial}{\partial x}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right] = \cos\theta \frac{\partial}{\partial r}\left[\frac{f(t - r/c_0)}{r} + \frac{g(t + r/c_0)}{r}\right]$$

Useful mathematical formulae

Spherical polar coordinates (r, θ, ψ)

Gradient

$$\nabla p' = \left(\frac{\partial p'}{\partial r}, \frac{1}{r}\frac{\partial p'}{\partial \theta}, \frac{1}{r\sin\theta}\frac{\partial p'}{\partial \psi}\right)$$

Divergence

$$\nabla \cdot \mathbf{v}' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 v_r'\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta v_\theta'\right) + \frac{1}{r\sin\theta}\frac{\partial v_\phi'}{\partial \psi}$$

Laplacian

$$\nabla^2 p' = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial p'}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial p'}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 p'}{\partial \psi^2}$$

Delta functions

Kronecker Delta

$$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases}$$

1D δ-function $\delta(x) = 0$ for $x \neq 0$ and $\int_{-\infty}^{\infty} \delta(ax-b) f(x) dx = f(b/a)/|a|$

3D δ-function $\delta(\mathbf{x}) = \delta(x_1)\delta(x_2)\delta(x_3)$

Convolution algebra

Convolution of $f(\mathbf{x})$ and $g(\mathbf{x})$

$$(f \star g)(\mathbf{x}) = \int_{-\infty}^{\infty} f(\mathbf{y}) g(\mathbf{x}-\mathbf{y}) d\mathbf{y}$$

Commutative properties

$$f \star g = g \star f$$

$$\frac{\partial}{\partial x_i}(f \star g)(\mathbf{x}) = f \star \frac{\partial g}{\partial x_i} = \frac{\partial f}{\partial x_i} \star g$$

Green's function

3D Green's function for wave equation

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2\right) g(\mathbf{x}, t | \mathbf{y}, \tau) = \delta(t-\tau)\delta(\mathbf{x}-\mathbf{y})$$

$$g(\mathbf{x}, t | \mathbf{y}, \tau) = \frac{\delta\left\{|\mathbf{x}-\mathbf{y}| - c_0(t-\tau)\right\}}{4\pi c_0 |\mathbf{x}-\mathbf{y}|}$$

Lighthill's Acoustic Analogy

Lighthill's equation

$$\left(\frac{\partial^2}{\partial t^2} - c_0^2 \nabla^2\right) \rho' = \frac{\partial^2 T_{ij}}{\partial x_i \partial x_j}.$$

For cold, isentropic, low Mach-number jets, T_{ij} can be approximated as:

$$T_{ij} = \rho_0 u_i u_j$$

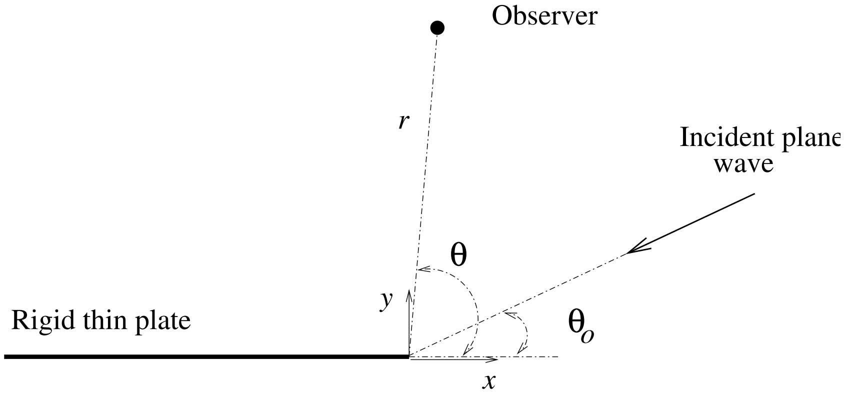

Figure 1: Geometry for edge scattering

Lighthill eight power law Acoustic power,

$$P_a \sim \frac{\rho_o d_j^2}{c_0^5} u_j^8,$$

where d_j and u_j are the jet exit diameter and velocity, respectively.

Refraction

Snells's law for determining a ray path is

$$\frac{\sin\theta}{c_0} = \text{constant} \ . \tag{1}$$

Diffraction

Field scattered by a sharp edge If the incident plane waves is

$$p_{\mathrm{i}}(\mathbf{x},t) = P_{\mathrm{inc}} \exp(\mathrm{i}\omega t + \mathrm{i}k_0 x \cos\theta_0 + \mathrm{i}k_0 y \sin\theta_0) \ , \tag{2}$$

then the diffracted pressure is

$$p_{\mathrm{d}} = P_{\mathrm{inc}} \left(\frac{2}{\pi k_0 r}\right)^{\frac{1}{2}} \frac{\sin(\theta_0/2)\sin(\theta/2)}{\cos\theta + \cos\theta_0} \exp\left(-\frac{\mathrm{i}\pi}{4} - \mathrm{i}k_0 r\right) \ . \tag{3}$$

In a cylindrical duct of radius a

The pressure field is given by

$$p'(\mathbf{x},t) = e^{i(\omega t + n\theta)} J_n(z_{mn} r/a)(Ae^{-ikx_3} + Be^{ikx_3}),$$

where z_{mn} is the mth zero of $\dot{J}_n(z)$ and $k = (k_0^2 - z_{mn}^2/a^2)^{1/2}$.

For large azimuthal wavenumber, n

$$z_{1n} \approx n + 1.85 n^{1/3}$$