EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 23 April 2019   2 to 3.40

Module 4A15

AEROACOUSTICS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4A15 Aeroacoustics data sheet (6 pages)
Engineering Data Book.

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An acoustic point source of frequency ω is located at the origin, and produces a field with unsteady velocity potential

$$\frac{A}{r}e^{i\omega(t-r/c_0)}$$

where A is a positive real constant and r is the distance of the observer from the origin. Assume that the background medium in which the sound propagates is homogeneous with a speed of sound c_0 and density ρ_0.

(a) Calculate the acoustic velocity and pressure generated by this source. [10%]

(b) Assuming that the observer is in the far field, determine the time-averaged intensity and power radiated by the source. [20%]

(c) A second source is placed at the point $(2a, 0, 0)$, and is identical to the first apart from a constant phase difference ψ, so that A in the source above can be replaced by $A\exp(i\psi)$. For the two sources together the amplitude of the acoustic pressure in the far field can be shown to be

$$\frac{2A\rho_0\omega}{r}\left|\cos\left(ka\cos\theta+\frac{\psi}{2}\right)\right|$$

where $k = \omega/c_0$ and θ is the angle between the observer vector and the first coordinate direction.

Calculate the total time-averaged power radiated from the two sources and show that the ratio of this quantity to that obtained in (b) above for a single source is

$$2\left[1+\frac{\sin(2ka)}{2ka}\cos\psi\right]$$

Hint:

$$\int_{-1}^{1}\cos^2(\psi/2+kau)\,du = 1+\frac{1}{2ka}\cos\psi\sin(2ka)$$

[50%]

(d) By using two values of ψ: 0 and π, explain the significance of the result in part (c) when $ka \gg 1$ and when $ka \ll 1$. [20%]

2 A vibrating string stretched from $(0,0,-L)$ to $(0,0,L)$ exerts a force per unit length $f(x_3,t)\,\mathbf{e_1}$ on the surrounding fluid, where $\mathbf{e_1}$ is the unit vector in the 1–direction and $|x_3| < L$. The string is vibrating at a frequency for which the source can be treated as spatially compact and thus acts like a point force of strength

$$F(t) = \int_{-L}^{L} f(x_3,t)dx_3$$

(a) Show that the far-field sound pressure field generated by the string is given by

$$p'(\mathbf{x},t) = -\frac{1}{4\pi}\frac{\partial}{\partial x_1}\left[\frac{F(t-r/c_0)}{r}\right]$$

where r is $|\mathbf{x}|$ and c_0 is the speed of sound in air. [55%]

(b) For

$$f(x_3,t) = \varepsilon\cos\left(\frac{\pi x_3}{2L}\right)e^{i\omega t}$$

where ε is a real constant, show that the far-field pressure is given by

$$p'(\mathbf{x},t) = \frac{i\omega\varepsilon L x_1}{r^2\pi^2 c_0}e^{i\omega(t-r/c_0)}$$

[25%]

(c) What is the main disadvantage of using a vibrating string to generate sound in a musical instrument such as the guitar? How can this be overcome? [20%]

3 An open rigid bottle shown in Fig.1 has a neck of cross-sectional area A and effective length l, and a bulb of volume V.

(a) State clearly the conditions under which $m'(t)$, the rate of mass flow out of the bottle, is related to $p'_1(t)$ the pressure perturbation at the neck opening and $p'_2(t)$ the pressure perturbation in the bulb by both

$$\dot{p}_2 = -\frac{c_0^2}{V} m'(t)$$

and

$$p'_2 - p'_1 = \frac{l}{A}\dot{m}$$

where c_0 is the speed of sound and the dot denotes a time derivative. Note that detailed calculations are not required. [30%]

(b) When these conditions hold, determine the relationship between $p'_1(t)$ and $m'(t)$ for fluctuations of frequency ω. [15%]

(c) Determine the sound power radiated from a bottle with the following parameters: $l = 5 \times 10^{-2}$ m; $A = 3 \times 10^{-4}$ m^2; $V = 10^{-3}$ m^3; $c_0 = 340$ m s^{-1}; $p'_1(t) = \hat{p}_1 e^{i\omega t}$; $\hat{p}_1 = 4$ N m^{-2}; $\omega = 800$ rad s^{-1}. The mean density of the air in the bottle and its surroundings is $\rho_0 = 1.2$ kg m^{-3}. [55%]

Hint: The sound pressure at a radial distance r from a monopole point source with rate of mass outflow $m'(t)$ is

$$p'(r,t) = \frac{\dot{m}(t - r/c_0)}{4\pi r}$$

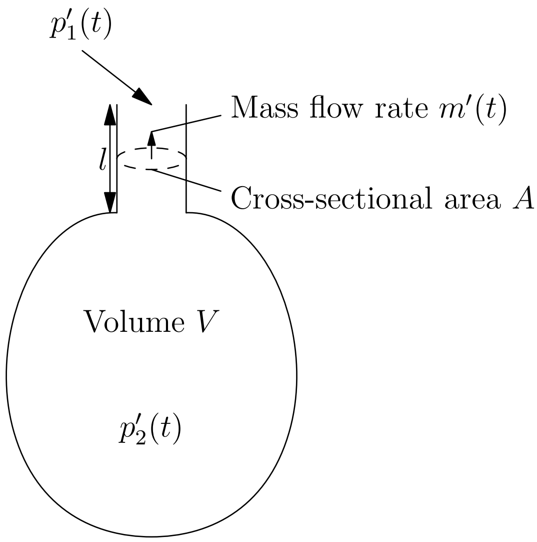

Fig. 1

4 In two dimensions a fluid occupies the half space $x > 0$ bounded by a wavy wall at $x = 0$. The maximum displacement of the wall from $x = 0$ is small compared to the wavelength. The wall moves along its length in such a way that the unsteady velocity in the x direction on $x = 0$ is given as

$$V_0 \exp(\mathrm{i}\omega t - \mathrm{i}k_2 y)$$

There are no incoming waves from infinity.

(a) Consider the case in which the fluid in $x > 0$ is uniform with sound speed c_0 and density ρ_0 everywhere. Determine an expression for the unsteady pressure in $x > 0$, being careful to distinguish between the cases $k_2 < \omega/c_0$ and $k_2 > \omega/c_0$. [40%]

(b) Now consider the case in which the fluid in $x > 0$ has sound speed $c_1 > c_0$ in the region $0 < x < L$ and sound speed c_0 in $x > L$. The density ρ_0 is uniform throughout $x > 0$. Explain the physical significance of the inequalities

$$c_0 < \omega/k_2 < c_1 \tag{1}$$

[20%]

(c) Determine expressions for the unsteady pressure in $x > L$ when condition (1) of part (b) is satisfied. [40%]

END OF PAPER