Version SJS/1.3

EGT3
ENGINEERING TRIPOS PART IIB

Friday 03 May 2019 9.30 to 11.10

Module 4B23

OPTICAL FIBRE COMMUNICATION

*Answer not more than **two** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4B23 Optical Fibre Communication formula sheet (2 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A step index optical fibre has a core radius $a = 5\ \mu$m with core refractive index $n_{co} = 1.4545$ and cladding refractive index $n_{cl} = 1.44$

(a) Explain the role of the normalised wavenumber V in the analysis of step index optical fibres [10%]

(b) List the four LP modes that are supported by this waveguide at 1550 nm and hence determine the maximum wavelength range over which these four LP modes are supported [30%]

(c) For two cores of radius a whose centres are separated by Δ, the approximate coupling coefficient κ between the two LP_{01} modes is given by

$$\kappa \approx \frac{V^{(2-\rho^2)}}{a^2 n_{cl} k_0}$$

where $\rho = \Delta/a$, $k_0 = 2\pi/\lambda$ and λ is the wavelength in vacuo.

(i) Using a coupled mode approach or otherwise, show that after a propagation distance z the proportion of power coupled from one core to the other if $\kappa z \ll 1$ is approximately $(\kappa z)^2$ and therefore that the crosstalk measured in decibels is $20\log_{10}(\kappa z)$. [25%]

(ii) Determine the minimum integer value of ρ that gives a power crosstalk of less than -60 dB after 1 km at a wavelength of λ=1550 nm [15%]

(iii) Assuming in a multicore optical fibre (with each core having $a = 5\mu$m, $n_{co} = 1.4545$, $n_{cl} = 1.44$) the crosstalk combines incoherently and any three adjacent cores form an equilateral triangle, estimate the maximum number of cores which could be employed in a fibre with an overall diameter of 125 μm if the worst case crosstalk should not exceed -60 dB per km. [20%]

2 The EDFA underpins modern optical communication systems.

(a) Briefly explain the following terms in relation to the operation of an EDFA:

(i) Pump laser [5%]

(ii) Gain [5%]

(iii) Amplified spontaneous emission [5%]

(iv) Noise figure [5%]

(b) An optical amplifier with gain G has an input signal power S_{in} and associated noise power N_{in}. Starting with the definition of the noise figure, obtain an expression for the noise figure of the optical amplifier as a function of the gain G, and hence state the minimum noise figure for a high gain optical amplifier. [20%]

(c) An optical transceiver transmits with a rectangular Nyquist spectrum, 31.5 GBd PDM-QPSK with a signal to noise ratio (SNR) of 25 dB and delivers an optical power of -2 dBm at a wavelength of 1550 nm. This signal then propagates through 100 km of fibre with a loss of 0.2 dB/km. The fibre may be considered linear for the power transmitted. Two EDFAs are available for a communication link. The first is a low gain amplifier with a 6 dB noise figure with a gain of 10 dB, whereas the second is a high gain amplifier and has a quantum limited noise figure of 3 dB with a gain of 20 dB.

(i) Calculate the SNR, if after propagation it is amplified by the high gain amplifier. [10%]

(ii) Calculate the SNR, if after propagation the loss is overcome by using two low gain amplifiers in cascade. [10%]

(iii) Calculate the SNR if the two low gain amplifiers are distributed, such that the first amplifier occurs after 40 km and the second after 80 km [10%]

(d) A Raman amplifier is a distributed amplifier, which can be considered as the limit $z \to 0$ of a cascade of optical amplifiers (3 dB noise figure), with the gain matched to compensate for the loss of the fibre between amplifiers. Assuming the same signal power spectral density as in part (c), and that the attenuation of the fibre is reduced to 0.15 dB/km, estimate the maximum distance, in a bandwidth of 5 THz, over which a capacity of 100 Tbit/s could be transmitted. You may assume the transceivers are ideal and approach Shannon capacity. [30%]

3 It is proposed to transmit a 1 TbE signal over 1000 km of PSCF optical fibre using 100 GBd PDM-64QAM. The transmitter employs digital signal processing to create a rectangular Nyquist spectrum, with the chromatic dispersion compensated digitally at the receiver. At the operating wavelength of 1550 nm, the optical fibre has an attenuation of 0.16 dB/km, dispersion coefficient of 21 ps/nm/km and an effective area of 150 μm^2.

(a) Calculate the minimum number of taps N_{CD} required to compensate the chromatic dispersion in the 100 GBd signal transmitted over the 1000 km link if the digital coherent receiver uses an oversampling rate of 8/7. [20%]

(b) Assuming appropriate frequency domain implementation, estimate the minimum power consumption required to realise digital chromatic dispersion compensation for the 100 GBd PDM-64QAM signal assuming the energy required to perform a complex multiply is 1 pJ. You may assume there are no technological restrictions regarding the FFT size. [35%]

(c) If the signal is amplified every 100 km by an optical amplifier with a noise figure of 6 dB and gain 16 dB, calculate the maximum available signal to noise ratio (SNR) at the receiver assuming the optimum launch power is used. [25%]

(d) Finally, the signal is amplified every L km by a total of N in-line optical amplifiers, each with noise figure 6 dB, such that $(N + 1)L = 1000$ with the gain of each amplifier exactly compensating for the loss of the preceding fibre span. Determine the minimum number of in-line optical amplifiers if the required SNR at the receiver is 20 dB. [20%]

END OF PAPER

Formula	**Notes**
$\langle S\rangle = \frac{1}{2}\frac{n}{\eta_0}\lvert E_0\rvert^2$	$\langle S\rangle$ - time averaged Poynting Vector, E_0 - complex electric field, $\eta_0 = \sqrt{\mu_0/\varepsilon_0} = 377\ \Omega$, n is refractive index.
$P(z) = P(0)\exp(-\alpha z)$	α in nepers/km, related to loss in terms of α_{dB}, the loss in dB/km via $\alpha \approx 0.23\alpha_{dB}$
$L_{eff} = [1-\exp(-\alpha L)]/\alpha$	Effective length L_{eff} associated with L and loss α
$v_p = \frac{\omega_0}{\beta} = \frac{c}{n}$	Phase velocity v_p, for electric field $E = E_0 e^{j(\omega_0 t-\beta z)}$, where $\beta = k_0 n$ and n is refractive index and $k_0 = 2\pi/\lambda$ (note λ always refers to the wavelength in vacuo). $c = 3\times 10^8$ m/s
$v_g = \frac{d\omega}{d\beta}$	Group velocity v_g of a modulated wave $e^{j(\omega t-\beta z)}$
$n_g = c/v_g$	Group refractive index $n_g = n + \omega\frac{dn}{d\omega} = n - \lambda\frac{dn}{d\lambda}$
$D = -\frac{2\pi c}{\lambda^2}\beta_2 = -\frac{\lambda}{c}\frac{d^2n}{d\lambda^2}$	Dispersion coefficient D: $\beta_2 = \frac{d^2\beta}{d\omega^2}$. For λ =1550 nm $D\ (ps/nm/km) = -0.78\times\beta_2(ps^2/km)$
$\Delta t = D\Delta\lambda L$	Dispersion Δt, with dispersion coefficient D, spectral width $\Delta\lambda$, over a distance L. For 1550 nm 100 GHz spectrum corresponds to 0.8 nm
$V = k_0 a\sqrt{n_{co}^2 - n_{cl}^2}$	Normalised wavenumber for step index fibre. Core radius a, core refractive index n_{co}, cladding refractive index n_{cl}
$J_{m-1}(V) = 0$	Cutoff criterion for the modes. LP_{mn} is n^{th} solution of $J_{m-1}(V) = 0$ where J_m is the m^{th} order Bessel function of the first kind
$F(r) = \exp\left(-\frac{r^2}{r_0^2}\right)$	Gaussian approximation for fundamental mode with mode field radius: $r_0^2 = \frac{a^2}{\ln V}$
$\eta = \frac{2\sigma_1\sigma_2}{\sigma_1^2+\sigma_2^2}\exp\left(-\frac{\Delta^2}{\sigma_1^2+\sigma_2^2}\right)$	Overlap integral between two normalised Gaussian fields separated by Δ with the mode field radii σ_1 and σ_2

First n zeros for $J_k(x)$

k \ n	1	2	3	4	5
0	2.405	5.520	8.654	11.792	14.931
1	3.832	7.016	10.173	13.324	16.471
2	5.136	8.417	11.620	14.796	17.960
3	6.380	9.761	13.015	16.223	19.409
4	7.588	11.065	14.373	17.616	20.827
5	8.771	12.339	15.700	18.980	22.218

Formula	**Notes**
$S_{3dB} = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & j \\ j & 1\end{bmatrix}$	Scattering matrix of a 3 dB coupler
$i = RP = R\|A\|^2$	Current in a photodiode, responsivity R, electric field amplitude A, optical power $P = \|A\|^2$
$\phi = \gamma P$	Kerr nonlinear phase shift: $\gamma = n_2 k_0 / A_{eff}$ where for optical fibres it can be assumed $n_2 = 2.6 \times 10^{-20}$ m^2/W
$\frac{\partial A}{\partial z} = j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2}$	Effect of dispersion in retarded frame of reference. With loss and nonlinearity becomes the NLSE $\frac{\partial A}{\partial z} = -\frac{\alpha}{2}A + j\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} - j\gamma\|A\|^2 A$
$P(\Delta\tau > x) \approx \frac{4}{\pi}\frac{x}{\langle\Delta\tau\rangle}\exp\left(-\frac{4x^2}{\pi\langle\Delta\tau\rangle^2}\right)$	Probability that with mean DGD $\langle\Delta\tau\rangle$ the instantaneous DGD $\Delta\tau$ exceeds x
$N_{NLI} = C_{NLI} G_{TX}^3$	Nonlinear noise power density for input PSD of G_{TX} with $C_{NLI} = \frac{8\gamma^2 L_{eff}^2 \alpha}{27\pi\|\beta_2\|}\ln\left(\frac{\|\beta_2\|}{\alpha}\pi^2 B^2\right)$
$N_{ASE} = 10^{NF/10} h\nu(G-1)$	Power spectral density for ASE amplifier with gain G and noise figure NF. $h = 6.634 \times 10^{-34}$ Js and for $\lambda \approx$1550 nm, $h\nu \approx 1.3 \times 10^{-19}$ J = 0.8 eV.
$G_{opt} = \sqrt[3]{\frac{N_{ASE}}{2C_{NLI}}}$	Optimum power spectral density
$N_q = 2h\nu P$	PSD for quantum noise
$N_{cm} = \frac{N\log_2(N) + N}{N - N_f + 1}$	Number of complex multiplications per sample for overlap and save implementation of a filter of length N_f using N point FFT (that in turn requires $0.5N\log_2 N$ complex multiplications)
$\boldsymbol{h} \coloneqq \boldsymbol{h} - \mu\frac{\partial\|\epsilon\|^2}{\partial\boldsymbol{h}^*}$	Stochastic gradient update for taps $\boldsymbol{h}$ with error ϵ and convergence parameter μ
$C = 1 - H_2(p_b)$	Capacity of binary symmetric channel where $H_2(p_b) = -(1-p_b)\log_2(1-p_b) - p_b\log_2 p_b$
$C = B\log_2(1 + SNR)$	Shannon capacity (for one polarisation), with bandwidth B and signal to noise ratio SNR

Numerical answers for 4B23 Optical Fibre Communication 2018/19 paper version 1.3

1. b) $1.253\ \mu m < \lambda < 1.680\ \mu m$ c) (ii) $\rho = 5$. (iii) 19 cores
2. c) (i) 23.5 dB (ii) 22.5 dB (iii) 24.7 dB d) 2180 km
3. a) 1920 samples b) 3.9 W c) 20.9 dB d) 9 amplifiers