Version CD/5

EGT3
ENGINEERING TRIPOS PART IIB

Date 27 April 2015 9:30 to 11

Module 4B5

NANOTECHNOLOGY

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain briefly what wave-particle duality is and describe how to calculate the wavelength of a particle and the momentum of a wave, within the context of quantum mechanics. [15%]

(b) Discuss the statement "light has both wave- and particle-like characteristics" with reference to key experimental evidence demonstrating both types of behaviour. Your answer should include sketches of the relevant experimental setups, and a discussion of how the results led to the formulation of quantum mechanics. [45%]

(c) In the Bohr model of the ground state of the hydrogen atom, the electron is considered to orbit the nucleus, which comprises a single proton, at a mean distance, r, and the electron's wavelength is assumed to be equal to the length of the circumference of that orbit, as shown in Fig. 1.

(i) Derive an expression for the total energy of the electron, taking into account the kinetic energy it possesses as it orbits the nucleus and the potential energy it has due to the electrostatic Coulomb force it experiences; [15%]

(ii) Using the expression derived in part (i) above, determine the value of the mean radius at which the electron's energy is a minimum; [15%]

(iii) Briefly discuss the shortcomings of the Bohr model. [10%]

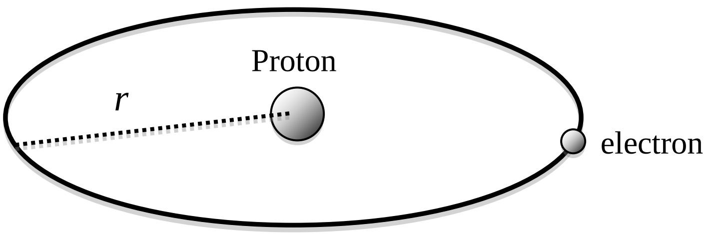

Fig. 1

2 The bonds between atoms in a material can be represented as springs, with a linear relationship between displacement, x, and restoring force for small displacements. From a classical (Newtonian) standpoint, an individual atom may be represented by a mass, m suspended from a spring of stiffness, k (as shown in Fig. 2), and will oscillate at the natural resonance frequency f_c:

$$f_c = \frac{1}{2\pi}\sqrt{\frac{k}{m}}\ .$$

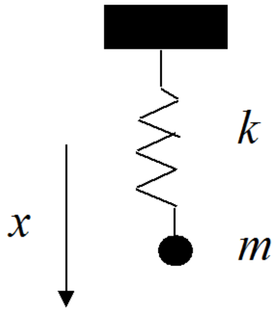

Fig. 2

(a) Write down the Schrödinger equation for this system, and solve it to determine the spectrum of energy levels, clearly stating any assumptions made. [40%]

(b) (i) Write down the normalised wavefunctions for the ground state and the first excited states of this system. Sketch these, and then sketch the corresponding probability density associated with both states. [20%]

(ii) Sketch the probability density associated with a classical harmonic oscillator, and discuss the differences between it and the quantum one. [15%]

(c) Heisenberg's uncertainty principle tells us that $\Delta x \Delta p > \hbar/2$, where x is position, p is momentum and $\hbar$ is Planck's constant divided by 2π. Using this result, determine the order of magnitude uncertainty in position of a Hydrogen atom of mass 1.67×10^{-27} kg, held by a bond of stiffness, k = 300 N/m, clearly stating any assumptions made. [25%]

3 (a) Quantum tunnelling is a fundamental phenomenon that occurs in many different contexts. With the aid of a sketch, briefly describe the process, and give one example of where it is prevalent. Why is classical mechanics unable to explain the phenomenon? [20%]

(b) (i) Describe the difference between the manufacture of a conventional diode and a resonant tunnelling diode; [10%]

(ii) Derive the approximate relationship between quantum transmission probability and current in an electronic device. [15%]

(c) For the resonant tunneling diode, draw the band diagrams (to include both the valence and conduction bands) corresponding to the following applied voltage biases:

(i) zero bias; [5%]

(ii) reverse bias; [5%]

(iii) small forward bias, for maximum current; [5%]

(iv) large forward bias. [5%]

(d) Draw the current-voltage characteristic of the resonant tunnelling diode, indicating each of the cases in part (c) above. [20%]

(e) Which property of the resonant tunnelling diode is important from an application standpoint? Describe a situation where it would be highly advantageous to include one in a circuit. [15%]

4 (a) Describe the principle of operation of the Atomic Force Microscope (AFM) in tapping-mode. Your answer should include a discussion of the forces involved, the particular choice of tapping frequency, the two main signals that are monitored and the circumstances under which tapping-mode is employed. [30%]

(b) Describe the conditions under which bond-level resolution may be achieved in AFM, and comment on how this compares against the resolution of the Scanning Tunneling Microscope. [20%]

(c) We have been given a polymer blend sample comprising two polymers, denoted by *A* and *B*, which are known to be hydrophilic and hydrophobic, respectively. In some areas on the surface, there are micron-scale regions of polymer *A* surrounded by polymer *B*, but without any associated change in topography, as indicated in Fig. 3. Devise an experiment using an AFM to:

(i) Distinguish between the two polymers, but without being able to tell which is which; [20%]

(ii) Uniquely determine which areas are polymer *A* and which are polymer *B*. [30%]

Your answer should include a brief discussion on the measurement principles, their complexities and any possible alternative measurement techniques that could be employed.

Fig. 3

5 (a) Derive the wavefunctions and the spectrum of allowed energy levels for electrons that are placed in an infinitely deep potential well of width L. [30%]

(b) For the case L = 10 nm and where the electron effective mass is 0.3 times the free electron mass:

(i) calculate the energy of the ground state and the first three excited states; [15%]

(ii) write down the wavefunctions for the ground state and the first excited state, and sketch the corresponding probability densities. [20%]

(c) Comment on the boundary conditions used to solve for these wavefunctions and on their physical meaning. [15%]

(d) Now consider the case where the well is not infinitely deep. Discuss how the energy levels and wavefunctions will change, and comment on the nature of the wavefunction *outside* the well. [20%]

END OF PAPER

Numerical answers.

1. (c) (ii) r = 23.9 pm

2. (c) 6 pm

5. (b) $E_n = 12.5n^2$ meV. n_1 = 12.5 meV, n_2 = 50 meV, n_3 = 112.5 meV, n_4 = 200 meV