EGT3
ENGINEERING TRIPOS PART IIB

Thursday 21 April 2016 9.30 to 11

Module 4B5

NANOTECHNOLOGY

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Electrons at an energy, E are incident from the left on a one-dimensional potential step of height V, which is 0.4 eV higher than E, as shown below in Fig. 1.

(a) By solving Schrödinger's time-independent equation, derive the single-particle wavefunctions for regions I and II, stating any assumptions made. Hence, calculate the reflection probability for this potential step. [40%]

(b) Now consider the case where the potential step has a finite length, d as shown below in Fig. 2. Write down the wavefunctions and sketch the probability densities in regions I, II and III. If d is 1 nm, *estimate* the amplitude of the transmitted wavefunction. [40%]

(c) What is the phenomenon in part (b) above called? Briefly describe an example of this phenomenon. [20%]

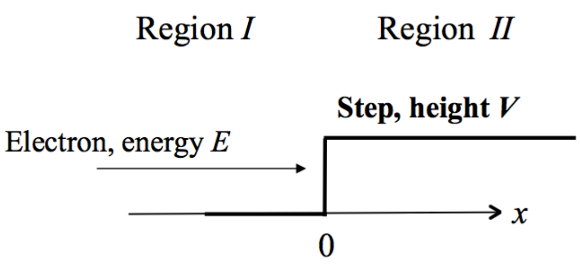

Fig. 1

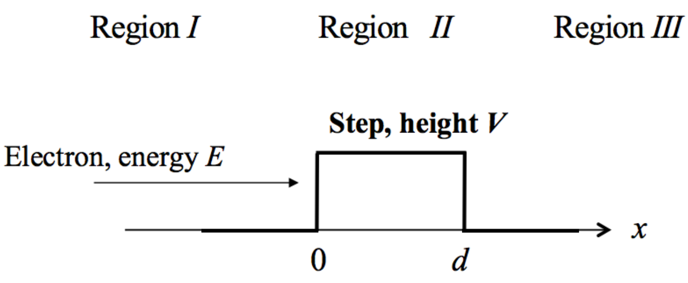

Fig. 2

2 Consider a heterostructure device with the conduction-band edge profile as shown below in Fig. 3. Within the central region, there are two bound states, labeled E_1 and E_2.

(a) Sketch what happens to the potential profile when a voltage is applied between both sides, such that electrons will flow from left to right. Indicate on your sketch the states that the conduction electrons flow from and into. [25%]

(b) Describe what happens to the transmission probability, T as the applied voltage is increased. By considering the relationship between T and current, sketch the form of the current vs. voltage characteristic for the device, labelling the important features. [25%]

(c) Sketch the wavefunctions for an electron travelling through this device for the case where the applied voltage is such that the conduction band edge on the left hand side

(i) is below E_1; [10%]

(ii) matches either E_1 or E_2; [10%]

(iii) is between E_2 and the top of the barrier; [10%]

(iv) is above the top of the barrier. [10%]

(d) Briefly describe how such a structure could be fabricated, with reference to the materials used and typical device dimensions. [10%]

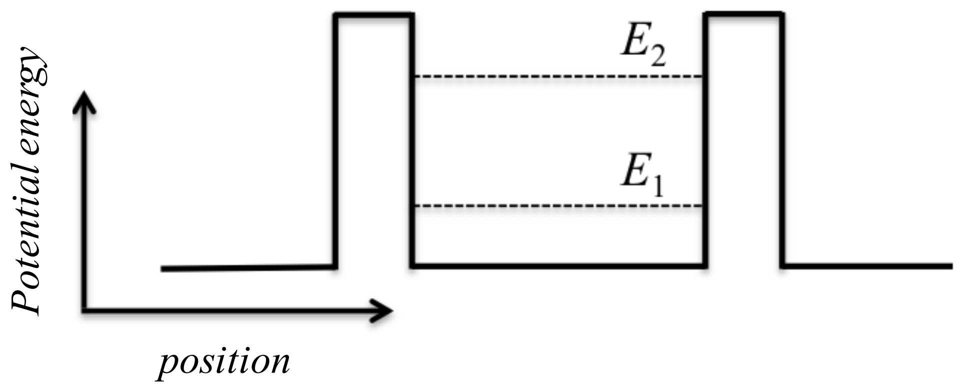

Fig. 3

3 (a) Describe, with the aid of a sketch, what happens to the energy levels when two quantum wells are brought into close enough proximity to each other that the wavefunctions for the electrons in each well start to overlap with the wavefunctions for the electrons in the other well. [20%]

(b) (i) State the assumptions used in the free-electron and the nearly-free electron models of electronic conduction. [10%]

(ii) Sketch the periodic Coulomb potential that electrons experience when moving in a crystal. [10%]

(iii) By considering the Coulomb potential, V and the electron wavefunctions, ψ in the nearly-free electron model to have the form

$$V(x) = V_0 + V_1 e^{i\frac{\pi x}{a}}$$

and

$$\psi(x) = (C_0 + C_1 e^{i\frac{\pi x}{a}})e^{ikx}$$

where k is the electron wavenumber and a is the atomic spacing, solve the corresponding Schrödinger equation to derive an expression from which the relationship between electron energy and momentum may be determined. Clearly state any assumptions made and outline the solution steps. [50%]

(c) Sketch the dispersion relation for a nearly-free electron, clearly labeling the salient features. [10%]

4 (a) Describe the principle of operation of the Scanning Tunneling Microscope (STM). What information about a sample can be obtained using an STM? [20%]

(b) Discuss the design considerations that must be taken into account when constructing an STM. [20%]

(c) Briefly discuss the relative merits of STM and Atomic Force Microscopy (AFM), in terms of ease of use, range of applicability, resolution and types of information that can be obtained. [20%]

(d) Recent years have seen significant advances in AFM technology, enabling measurement types previously impossible. Discuss this statement with the use of examples. [20%]

(e) An STM tip is positioned 0.7 nm above a metal surface, which has a work-function of 4 eV. Calculate the maximum allowed deviation from this distance if the tunneling current is to be maintained to within 1% of its nominal value. [20%]

(TURN OVER

5 (a) Define *Nanotechnology*. Discuss how it is being employed in our everyday lives, with at least three examples. [25%]

(b) With reference to the relevant lengthscales, briefly discuss why the physical properties of objects differ from their bulk values once the object's dimensions are on the nanometre scale. [20%]

(c) Briefly discuss, with appropriate use of examples, the relevance of quantum mechanics to nanotechnology. [10%]

(d) The interaction of an electron with a semiconductor heterostructure is illustrated in Fig. 4, where panels (i)-(iv) are taken at successive time intervals. The electron is incident from the left, and has an energy, E. Discuss how the wavefunction describing the electron evolves during the timeframe illustrated here. [45%]

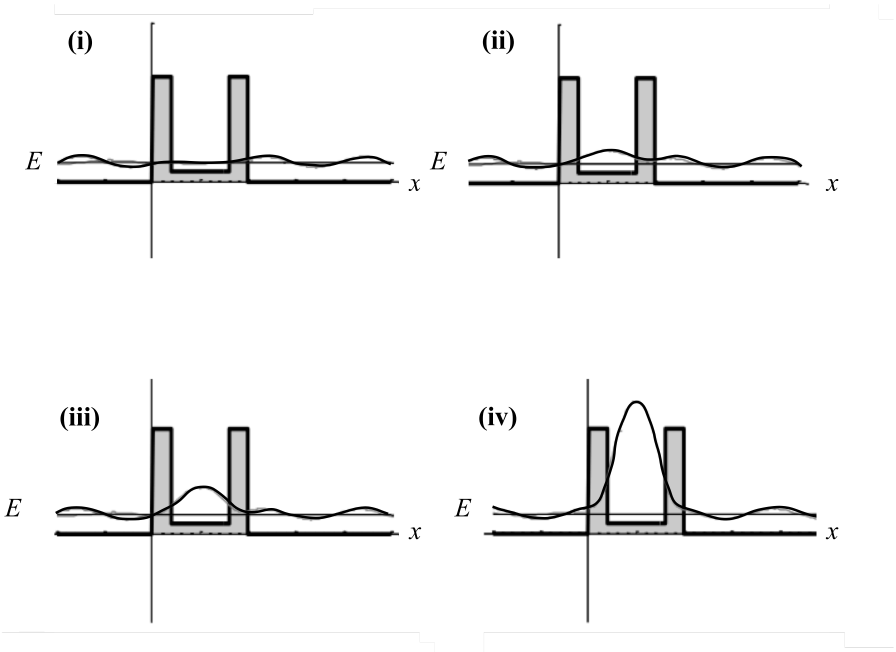

Fig. 4

END OF PAPER

4B5 solutions 2016

Short answers

1.

(a) $R = 1$

(b) $T = 1.5\text{x}10^{-3}$