Version AAS/4

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 21 April 2015 9.30 to 11

Module 4C15

MEMS DESIGN

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C15 MEMS Design data sheet (3 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 A smooth, rigid planar substrate is covered by a Self Assembled Monolayer (SAM) as illustrated in Fig. 1. The molecular chain length is h and the molecules can be considered to be stacked perpendicularly to the surface. There are n molecules of the SAM per unit area of the surface where n is a large number.

The layer is indented to a depth $\delta_1 (< h)$ by the probe of an Atomic Force Microscope. The tip of the probe can be thought of as a rigid sphere of radius R and the applied normal force is P_1 as illustrated in Fig. 1(a). The indentation is elastic and each of the SAM molecules can be considered to be acting as an independent linear spring of stiffness k. The contact patch is of radius a.

(a) If $a \ll R$, so that the shape of the tip can be taken to be such that $\delta_1 = a^2 / 2R$, show that the effective pressure distribution $p(r)$ between the sphere and the SAM can be expressed as

$$p(r) \approx \frac{nk}{2R}\left(a^2 - r^2\right)$$ [20%]

(b) Hence obtain an expression relating the normal load P_1 on the sphere to the values of n, k, R and δ_1. [20%]

(c) Sketch a curve illustrating the way the load on the tip varies with indentation depth. When the indentation is δ_1 how is U_{E1}, the energy stored elastically in the layer, related to n, k, R and a? [20%]

(d) Van der Waal forces lead to the ends of the molecules of the SAM adhering to the surface of the probe tip so that the contact patch remains the same size, i.e. of radius a, even when the tip is withdrawn through $\Delta\delta$, so to a displacement $\delta < \delta_1$. This is accompanied by a reduction in the load to $P = P_1 - \Delta P$ as illustrated in Fig. 1(b). Indicate how this change plots on your graph of load versus displacement. [15%]

(e) Hence obtain an expression for the stored elastic energy U_E in this new configuration in terms of n, k, R, a and P. [15%]

(f) Indicate qualitatively how this argument could be continued to establish an estimate for the “pull-off” force for this adhesive situation. [10%]

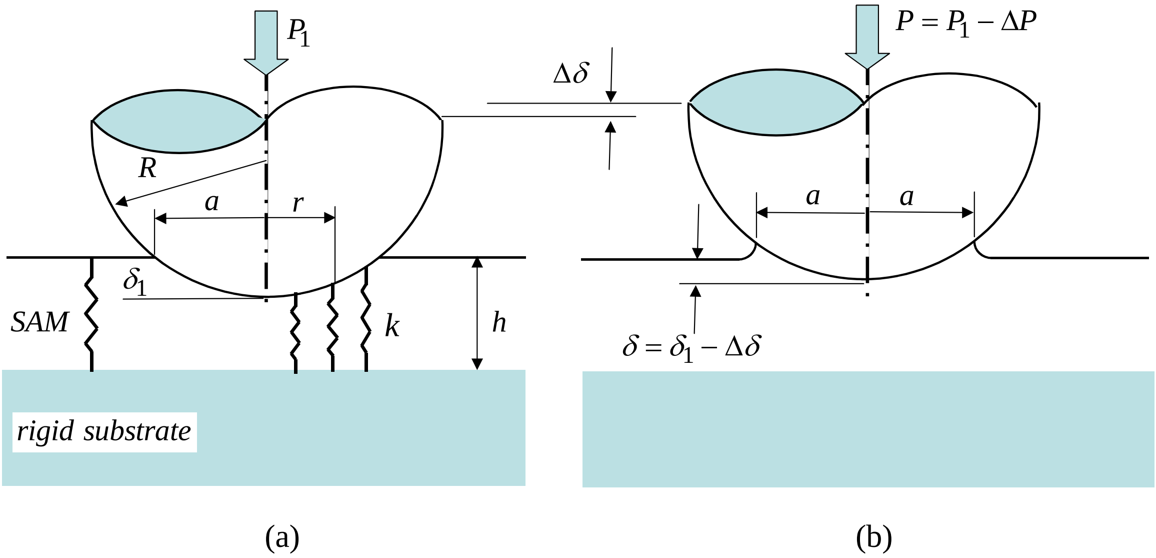

Fig. 1

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2 A torsional actuator comprising of a torsional spring of value k_α is shown in cross-section in Fig. 2. The actuator comprises an upper plate of length 2*l*, width *b* and thickness *t*. The actuator is operated by applying a voltage *V* with respect to a fully overlapping fixed electrode of width *b* located below the upper plate as shown. The nominal spacing between the electrode and the plate with no voltage applied is *d*.

(a) Derive an expression for the electrostatic torque generated for a given applied voltage *V* and a corresponding tilt angle α. [40%]

(b) Assume that the lower electrode spans the entire length of the actuator i.e. a_1=0 and a_2 =*l*. Estimate the equilibrium tilt angle α for a given operating voltage *V* assuming that the tilt angle is small. [20%]

(c) Write down the equations (but do not solve) to derive an expression for the pull-in voltage as a function of system parameters assuming a_1=0 and a_2 =*l*. [20%]

(d) The critical angle at which pull-in occurs for part (c) above is given by $\alpha_{PI} = 0.44d/l$. Obtain an expression for the corresponding pull-in voltage. [20%]

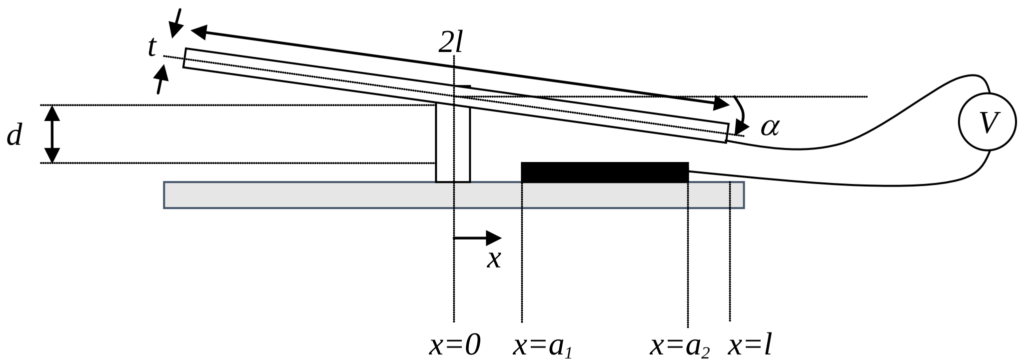

Fig. 2

3 A z-axis polysilicon surface-micromachined gyroscope comprises of a single suspended proof mass designed to be compliant along two orthogonal directions in the fabrication plane. The proof mass is set into motion along the drive axis using comb-drive actuators and the motion of the mass is detected using a differential parallel-plate sensing arrangement. The proof mass is $m=10^{-9}$ kg and the spring constants along the drive direction and sense directions are nominally set to 10 N m^{-1} and 12.1 N m^{-1} respectively. The polysilicon structural thickness is 10 μm and the electrode gap spacing is 1 μm. The device is hermetically packaged just under atmospheric pressure resulting in a Quality factor of 10 for resonant motion along the drive and sense directions. The number of comb finger gaps is N=1000 while the number of differential sense electrode cells is 50.

(a) Estimate the amplitude of motion along the drive axis if an ac voltage of 1 V and a dc voltage of 10 V are applied between the comb drive actuators and the proof mass for forcing at resonance. [30%]

(b) For the conditions outlined in (a) above, estimate the magnitude of the resulting sense mode displacement and the fractional change in sense capacitance for an applied rotation rate of 10 rad s^{-1} about the z-axis assuming no tuning of the resonant frequencies. [20%]

(c) Estimate the thermo-mechanical noise equivalent rotation rate at room temperature T = 300 K. [30%]

(d) The sense mode resonant frequency is tuned by applying a dc voltage between the sense electrodes and the proof mass. Estimate the magnitude of the dc voltage required to be applied to achieve mode-matched conditions if each sense electrode is 100 μm long. [20%]

4 A microfluidic device is shown in top view in Fig. 3 comprising of a glass channel where an electrolyte solution is pumped from Port 1 to Port 2. The length of the channel is 5 mm and the channel has a height of 200 μm throughout. The cross-sectional width of the microchannel has a value of 100 μm apart from a region 1 mm in length in between the two ports where the channel is constricted to a width of 10 μm as shown. The physical dimensions of the two ports may be neglected in the analysis.

(a) The relationship between volumetric flow rate Q and the uniform pressure gradient K for steady laminar pressure-driven flow through a pipe of rectangular cross-section is given by

$$Q = \frac{hw^3K}{12\eta}$$

where h is the channel height, w is the channel width, and η is the viscosity of the solution at room temperature (10^{-3} Pa s). Calculate the volumetric flow rate and the time taken to pump 1 μL of the solution from Port 1 to Port 2 of the device shown in Fig. 3 for a pressure drop of 10 kPa between Port 1 and Port 2. [40%]

(b) The device is now configured in a format such that the solution is pumped between Ports 1 and 2 using electro-osmosis. The resulting flow profile may be assumed to be consistent with plug flow such that the peak flow velocity (U_0) can be written in terms of the device parameters as

$$U_0 = -\frac{\varepsilon\zeta}{\eta}E_x$$

where ε is the dielectric constant of the solution, ζ is the associated zeta potential and E_x is the applied electric field. Assume that $\varepsilon = 80\varepsilon_0$, $\zeta = -100\,\text{mV}$, $\eta = 10^{-3}$ Pa s and a voltage of 70 V is applied across Ports 1 and 2 whose physical dimensions may be neglected in the analysis.

(i) Estimate the time taken for a plug of solution to travel from Port 1 to the point where the channel constriction begins i.e. distance l_1. [20%]

(ii) The device is used to separate two molecules under flow using capillary electrophoresis. The electrophoretic mobilities of the two molecular species are 2×10^{-8} m^2 V^{-1} s^{-1} and 3×10^{-8} m^2 V^{-1} s^{-1} respectively. Estimate their relative separation distance before the leading species enters the constriction and after the trailing species exits the channel constriction. [20%]

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(iii) Discuss the device parameters that should be optimised to obtain a good separation. [20%]

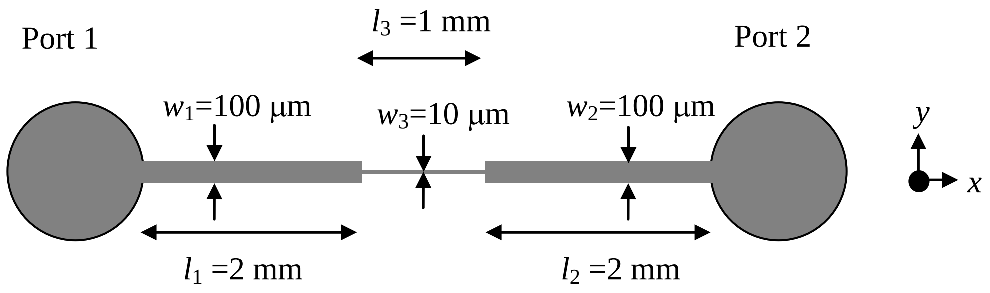

Fig. 3

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4C15 MEMS DESIGN

NUMERICAL ANSWERS

Q3

(a) 0.89 μm

(b) 0.38 nm, 0.76 x 10^{-3}

(c) 2.4 x 10-3 rad /s-rt-Hz

(d) 1.54 V

Q4

(a) 6 s

(b) (i) 5.65 s

(b) (ii) 3.97s, 198.5 μm; 4.63s, 233 μm