Version AEM/final

EGT3
ENGINEERING TRIPOS PART IIB

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27 April 2017 2 to 3.30

Module 4C2

DESIGNING WITH COMPOSITES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: Module 4C2 Designing with Composites data sheet (6 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 (a) Explain what is meant by "a *balanced* laminate". Show that a $\pm\phi$ "angle-ply" laminate, with a ply thickness $t/2$, is balanced when loaded at an angle equally inclined to the $+\phi$ and $-\phi$ plies. [30%]

(b) A small aircraft is being designed and an angle-ply fibre-glass polyester composite is being used for the fuselage. The fuselage, which will approximate to a thin-walled cylinder with closed ends of outside diameter D = 2 m is subjected to an internal pressure P and a bending moment M as shown in Fig. 1. The fuselage is produced by filament winding at $\pm 45°$ to the longitudinal axis of the fuselage. A total of 100 layers of +45° fibres and 100 layers of $-45°$ fibres are laid down, giving a total wall thickness t. Each layer has the following properties: $E_2 = 0.2E_1$, $G_{12} = 0.09E_1$ and $\nu_{12} = 0.3$.

(i) Find the stiffness matrix [Q] for each ply in the principal material axes (1, 2) in terms of E_1. [15%]

(ii) Find the extensional stiffness matrix [A] for the wall material in terms of E_1 and t, in axes (x,y) aligned with those of the cylinder. [25%]

(iii) Find the strains (ε_x, ε_y, γ_{xy}) induced in the tensile outer surface of the fuselage when it is subjected to an internal pressure of 0.06 MPa and a bending moment of 500 kN m. [30%]

[The second moment of area of a thin-walled cylinder is given by $\pi R^3 t$.]

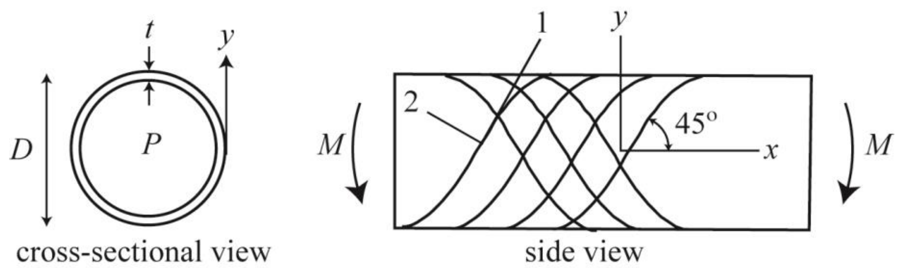

Fig. 1

2 Table 1 gives the stiffness properties of three laminates made from four plies of a CFRP prepreg material, where the laminate x and y axes coincide with the 0° and 90° directions respectively. Table 1 also includes the measured first-ply failure stress for tensile loading along the laminate x direction.

(a) Show that the given failure stresses are consistent with lamina failure strains of $\varepsilon_L^+ = 0.010$, $\varepsilon_T^+ = 0.0050$, $\varepsilon_L^- = 0.020$ and $\varepsilon_T^- = 0.010$. What are the corresponding expected failure modes for each of the laminates? [20%]

(b) Use the failure strains given in part (a) to predict the magnitude of equal tensile stresses applied in the x and y directions (i.e. a biaxial loading) which would cause first ply failure for each of the three laminates. [25%]

(c) For the cross-ply laminate given in Table 1, use the lamina failure stresses given on the datasheet for AS/3501 to predict the biaxial tensile stress loading, as per part (b), which would cause failure according to the Tsai-Hill failure criterion. [35%]

(d) Describe suitable tests to measure the uniaxial and biaxial tensile strength of such laminates. [20%]

Laminate	$[0_4]$	$[0,90]_s$	$[90_4]$
E_x (GPa)	138	74	9.0
E_y (GPa)	9.0	74	138
ν_{xy} (−)	0.30	0.037	0.020
Failure stress (MPa)	1400	370	45

Table 1

3 (a) Derive an expression for the compressive strength associated with plastic microbuckling of a unidirectional composite with initial fibre misalignment angle ϕ_0 and matrix shear yield strength τ_y. Detail any assumptions made in your derivation. [15%]

(b) Describe, with the aid of a sketch, the pultrusion process. [15%]

(c) Explain why the compressive strength of pultruded unidirectional CFRP might be significantly greater than that of equivalent moulded material. [10%]

(d) Discuss the factors that contribute to the increasing use of GFRP pultruded composites for bridge construction. [20%]

(e) Figure 2 illustrates a hollow square-section GFRP composite beam of uniform layup and wall thickness t. The side length d of the section, which can be assumed to be large compared with t, equals 100 mm. The beam of length 3 m is mounted as a cantilever, rigidly fixed to a wall. A transverse load F = 30 N and a torsional load Q = 80 N m are applied simultaneously to the free end of the cantilever. Use the carpet plots provided in Fig. 3 to select an appropriate ply mix and thickness for a laminate containing a mixture of 0°, 90° and ±45° plies which minimise the mass of the beam, while ensuring that the end deflection is less than 2 mm and the end rotation is less than 0.2° under the applied loading. [40%]

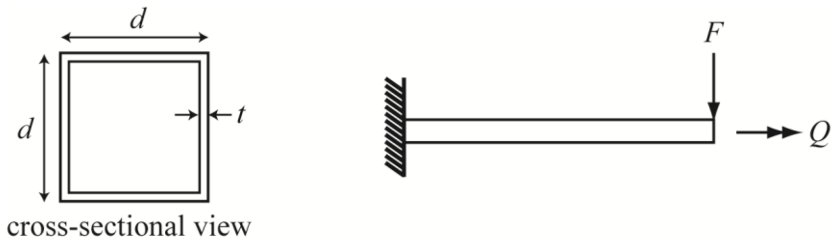

Fig. 2

(cont.

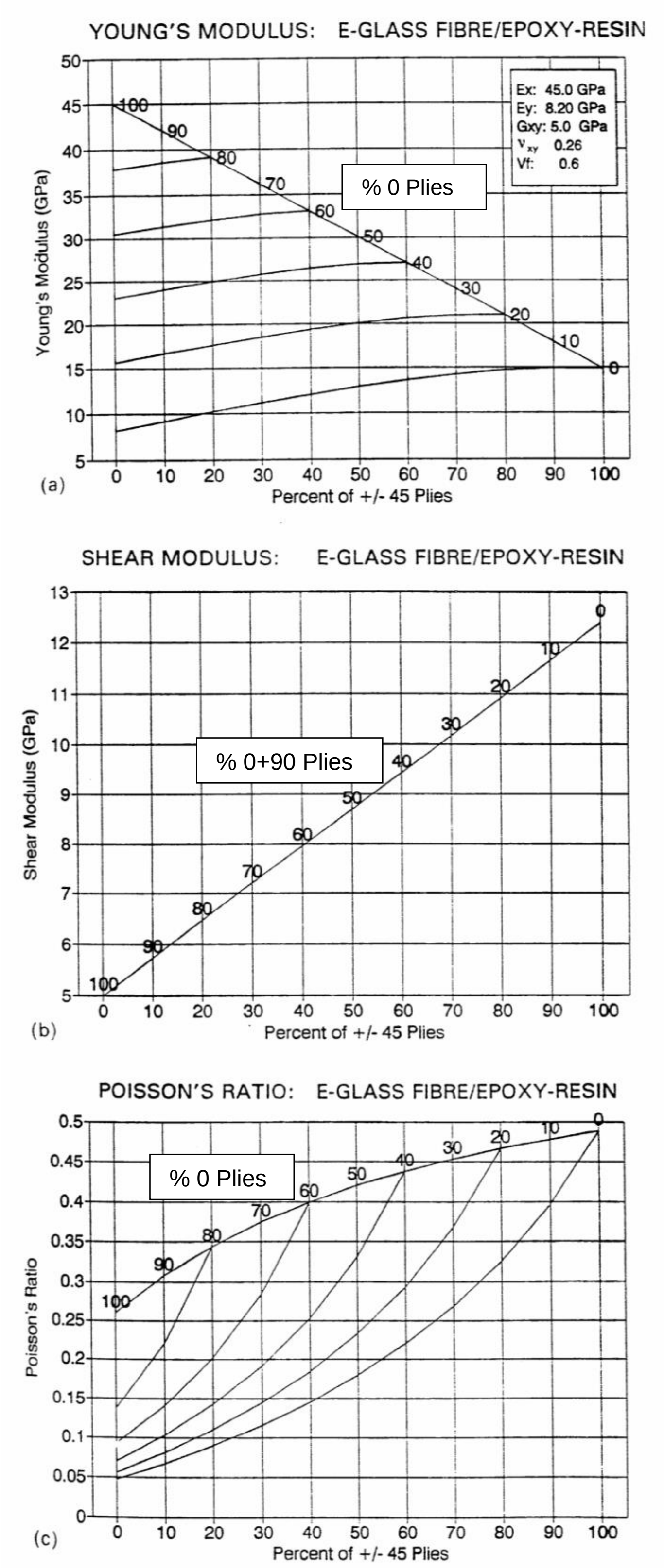
YOUNG'S MODULUS: E-GLASS FIBRE/EPOXY-RESIN
Ex: 45.0 GPa
Ey: 8.20 GPa
Gxy: 5.0 GPa
ν_{xy} 0.26
Vf: 0.6
% 0 Plies
Young's Modulus (GPa)
(a)
Percent of +/- 45 Plies
SHEAR MODULUS: E-GLASS FIBRE/EPOXY-RESIN
% 0+90 Plies
Shear Modulus (GPa)
(b)
Percent of +/- 45 Plies
POISSON'S RATIO: E-GLASS FIBRE/EPOXY-RESIN
% 0 Plies
Poisson's Ratio
(c)
Percent of +/- 45 Plies

Fig. 3

4 (a) Discuss the design and manufacture of composite garden furniture. Include the following points in your discussion: material and layup, manufacturing route, structural design and any other relevant aspects. [45%]

(b) Figure 4 illustrates a semi-circular unidirectional composite beam of radius *R*, thickness *t* and depth *b* (out of the plane of the figure) subjected to end loads *P*. Stress concentration effects at the loading points can be neglected.

(i) Derive an expression for the maximum through-thickness stress in the beam in terms of *P*, *R*, *b* and *t*. [20%]

(ii) Predict the load *P* which will cause failure of a beam made of AS/3501 (data on datasheet) with *R* = 800 mm, *b* = 60 mm and *t* = 20 mm. Detail any assumptions made. [20%]

(c) Discuss when through-thickness stresses in composites are of concern for predicting failure. [15%]

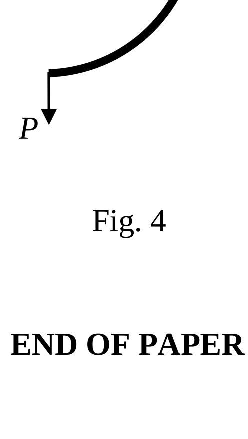

Fig. 4

END OF PAPER

Answers

1. (b)(i) $[Q] = E_1 \begin{bmatrix} 1.02 & 0.06 & 0 \\ 0.06 & 0.20 & 0 \\ 0 & 0 & 0.09 \end{bmatrix}$

(b)(ii) $[A] = tE_1 \begin{bmatrix} 0.43 & 0.25 & 0 \\ 0.25 & 0.43 & 0 \\ 0 & 0 & 0.28 \end{bmatrix}$

(b)(iii) $\begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix} = \frac{1}{tE_1} \begin{pmatrix} 5.4\times10^{-4} \\ -1.8\times10^{-4} \\ 0 \end{pmatrix}$

2. (b) $[0_4]$ $\sigma = 46$ MPa;
$[0,90]_s$ $\sigma = 380$ MPa;
$[90_4]$ $\sigma = 46$ MPa.

(c) $\sigma = 290$ MPa

4. (b)(i) $\sigma_z = \dfrac{3P}{2bt}$, (b)(ii) $P = 5860$ N