Version MPFS/3a

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 30 April 2019 9.30 to 11.10

Module 4C2

DESIGNING WITH COMPOSITES

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C2 Designing with Composites data sheet (6 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Consider a lamina of aligned continuous carbon fibres (E_f = 232 GPa) in a matrix of epoxy resin (E_m = 3.0 GPa), where the volume fraction of fibres is 60%.

(a) Estimate the axial and transverse stiffnesses for the lamina. State any assumptions or approximations made. [15%]

(b) Calculate the stiffness matrix $[Q]$ for the lamina in the principal material axes $(1, 2)$. [Additional laminate material properties: $\nu_{12} = 0.30, G_{12} = 6.92$ GPa.] [15%]

(c) The wall of a spherical vessel is made of the lamina described above, and the fibres have been wound so that at every point it approximates to a laminate stacked in the sequence $[(-60/0/60)_8]_s$ with respect to orthogonal local reference axes. Each lamina has a thickness of 250 μm, and the vessel diameter is 2 m.

(i) Calculate the laminate extensional stiffness matrix $[A]$ and the laminate coupling stiffness matrix $[B]$. Comment on their form. [50%]

(ii) Calculate the strains induced in the laminate when the vessel is internally pressurised to 10 bar. Hence, prove that the shape will remain spherical. [20%]

2 (a) Explain how strain allowables can be used to predict failure of composite laminates. What factors govern the values of strain allowables given for the composites in Table 1 of the data sheet? [20%]

(b) A circular tubular drive shaft of radius R = 150 mm and wall thickness t = 2 mm is subject to a torsional load Q and a bending moment M. The tube is made using GFRP laminae of thickness 0.125 mm with plies orientated at angles of 0, 90 and 45° to the axis of the tube. Use the carpet plots of Figure 1 to identify an appropriate layup which:

(i) maximises the torsional stiffness of the tube;

(ii) maximises the bending stiffness of the tube;

(iii) maximises the strength of the tube when only the torsional load Q is applied;

(iv) maximises the strength of the tube when equal torsional and bending loads are applied (i.e. $Q = M$). [60%]

(c) What additional factors would you consider to finalise the design of the commercial product? [20%]

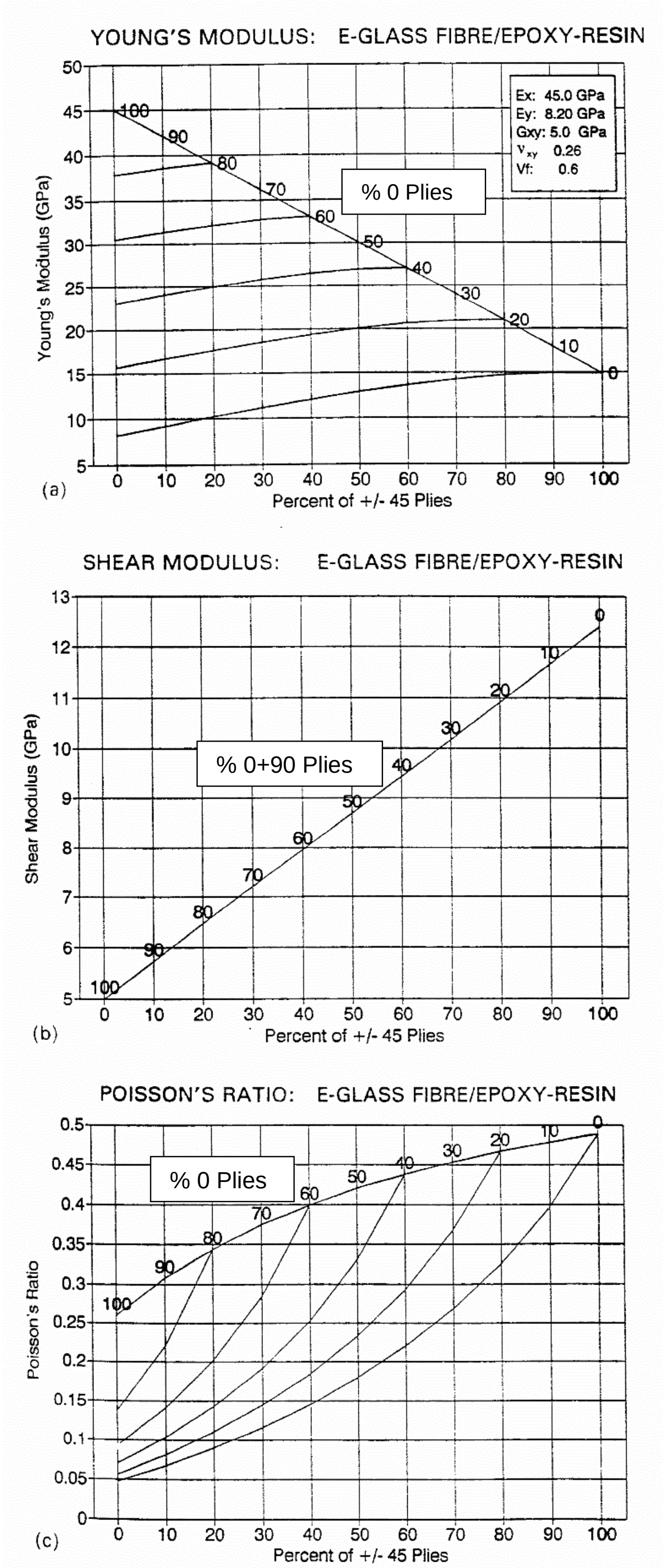


Fig. 1

(TURN OVER

3 (a) Describe and discuss different failure criteria for composite laminates. [25%]

(b) A unidirectional laminate is loaded bi-axially with a tensile stress $\sigma_x = \sigma$ and a compressive stress $\sigma_y = -\sigma$. The fibre direction is orientated at an angle ϕ relative to the x direction, as illustrated in Fig. 2. The laminate has the following failure stresses: $s_L^+ = s_L^- = 20\Sigma$, $s_{LT} = 3\Sigma$, $s_T^- = 2\Sigma$, $s_T^+ = \Sigma$, where Σ is a constant representative strength.

(i) Show that failure is predicted by the Tsai-Hill failure criterion to occur at a stress $\sigma < 2\Sigma$ for $-5.4° < \phi < 5.4°$. [40%]

(ii) Estimate the range of values of ϕ for which shear failure is expected to be the critical mechanism. [35%]

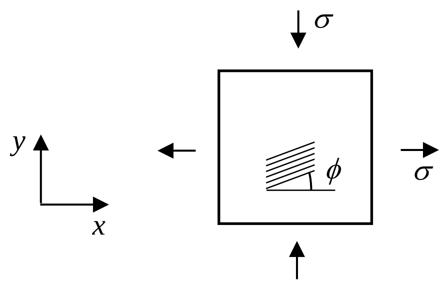


Fig. 2

4 (a) Outline a micromechanical model to predict toughness for cracking transverse to the fibre direction in long-fibre composites. [25%]

(b) Describe appropriate test methods to assess impact tolerance of composite laminates in aerospace applications. What factors in the laminate design and manufacture are likely to have the most effect on the results of these tests? [25%]

(c) Describe with sketches the compression moulding and injection moulding processes for composites. [15%]

(d) Discuss factors likely to affect the choice of material and manufacturing process for composite parts, illustrating your answer with reference to the following composite components: a leisure boat, the mast of a racing yacht, the air deflector on top of a truck cab, an airbag housing in a car. [35%]

END OF PAPER

ENGINEERING TRIPOS PART II B

Module 4C2 – Designing with Composites

DATA SHEET

The in-plane compliance matrix [S] for a transversely isotropic lamina is defined by

$$\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [S] \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} \qquad \text{where } [S] = \begin{bmatrix} 1/E_1 & -\nu_{21}/E_2 & 0 \\ -\nu_{12}/E_1 & 1/E_2 & 0 \\ 0 & 0 & 1/G_{12} \end{bmatrix}$$

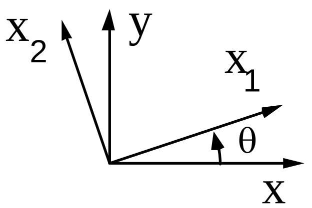
[S] is symmetric, giving $\nu_{12}/E_1 = \nu_{21}/E_2$. The compliance relation can be inverted to give

$$\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix}$$

$$\text{where } \begin{aligned} Q_{11} &= E_1/(1-\nu_{12}\nu_{21}) \\ Q_{22} &= E_2/(1-\nu_{12}\nu_{21}) \\ Q_{12} &= \nu_{12}E_2/(1-\nu_{12}\nu_{21}) \\ Q_{66} &= G_{12} \end{aligned}$$

Rotation of co-ordinates

Assume the principal material directions (x_1, x_2) are rotated anti-clockwise by an angle θ, with respect to the (x, y) axes.



Then, $\begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_{12} \end{pmatrix} = [T] \begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}$ and $\begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{pmatrix} = [T]^{-T} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$

$$\text{where } [\mathrm{T}] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix}$$

$$\text{and } [\mathrm{T}]^{-T} = \begin{bmatrix} \cos^2\theta & \sin^2\theta & \sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -\sin\theta\cos\theta \\ -2\sin\theta\cos\theta & 2\sin\theta\cos\theta & (\cos^2\theta - \sin^2\theta) \end{bmatrix}$$

The stiffness matrix $[Q]$ transforms in a related manner to the matrix $[\overline{Q}]$ when the axes are rotated from (x_1, x_2) to (x, y)

$$[\overline{Q}] = [T]^{-1}[Q][T]^{-T}$$

In component form,

$$[\overline{Q}] = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \text{ where } \begin{aligned} \overline{Q}_{11} &= Q_{11}c^4 + Q_{22}s^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\ \overline{Q}_{12} &= \left(Q_{11} + Q_{22} - 4Q_{66}\right)s^2c^2 + Q_{12}\left(c^4 + s^4\right) \\ \overline{Q}_{22} &= Q_{11}s^4 + Q_{22}c^4 + 2\left(Q_{12} + 2Q_{66}\right)s^2c^2 \\ \overline{Q}_{16} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)sc^3 - \left(Q_{22} - Q_{12} - 2Q_{66}\right)s^3c \\ \overline{Q}_{26} &= \left(Q_{11} - Q_{12} - 2Q_{66}\right)s^3c - \left(Q_{22} - Q_{12} - 2Q_{66}\right)sc^3 \\ \overline{Q}_{66} &= \left(Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}\right)s^2c^2 + Q_{66}\left(s^4 + c^4\right) \end{aligned}$$

with $c = \cos\theta, \quad s = \sin\theta$

The compliance matrix $[S] \equiv [Q]^{-1}$ transforms to $[\overline{S}] \equiv [\overline{Q}]^{-1}$ under a rotation of co-ordinates by θ from (x_1, x_2) to (x, y), as

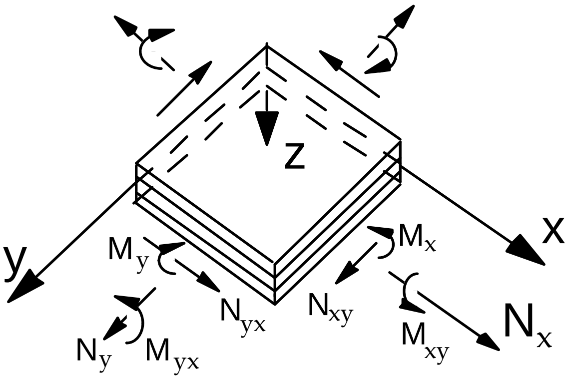
$$[\overline{S}] = [T]^T [S][T]$$

and in component form,

$$\begin{aligned} \overline{S}_{11} &= S_{11}c^4 + S_{22}s^4 + \left(2S_{12} + S_{66}\right)s^2c^2 \\ \overline{S}_{12} &= S_{12}\left(c^4 + s^4\right) + \left(S_{11} + S_{22} - S_{66}\right)s^2c^2 \\ \overline{S}_{22} &= S_{11}s^4 + S_{22}c^4 + \left(2S_{12} + S_{66}\right)s^2c^2 \\ \overline{S}_{16} &= \left(2S_{11} - 2S_{12} - S_{66}\right)sc^3 - \left(2S_{22} - 2S_{12} - S_{66}\right)s^3c \\ \overline{S}_{26} &= \left(2S_{11} - 2S_{12} - S_{66}\right)s^3c - \left(2S_{22} - 2S_{12} - S_{66}\right)sc^3 \\ \overline{S}_{66} &= \left(4S_{11} + 4S_{22} - 8S_{12} - 2S_{66}\right)s^2c^2 + S_{66}\left(c^4 + s^4\right) \end{aligned}$$

with $c = \cos\theta, \quad s = \sin\theta$

Laminate Plate Theory



Consider a plate subjected to stretching of the mid-plane by $\left(\varepsilon_x^o, \varepsilon_y^o, \varepsilon_{xy}^o\right)^T$ and to a curvature $\left(\kappa_x, \kappa_y, \kappa_{xy}\right)^T$. The stress resultants $\left(N_x, N_y, N_{xy}\right)^T$ and bending moment per unit length $\left(M_x, M_y, M_{xy}\right)^T$ are given by

$$\begin{pmatrix} N \\ \cdots \\ M \end{pmatrix} = \begin{bmatrix} A & \vdots & B \\ \cdots & . & \cdots \\ B & \vdots & D \end{bmatrix} \begin{pmatrix} \varepsilon^o \\ \cdots \\ \kappa \end{pmatrix}$$

In component form, we have,

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \gamma_{xy}^o \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix}$$

where the laminate extensional stiffness, A_{ij}, is given by:

$$A_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k dz = \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right)$$

the laminate coupling stiffnesses is given by

$$B_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k z dz = \frac{1}{2} \sum_{k=1}^{n} \left(\bar{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right)$$

and the laminate bending stiffness are given by:

$$D_{ij} = \int_{-t/2}^{t/2} \left(\bar{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

with the subscripts i, j = 1, 2 or 6.

Here,

n = number of laminae
t = laminate thickness
$z_{k\text{-}1}$ = distance from middle surface to the top surface of the *k-th* lamina
z_k = distance from middle surface to the bottom surface of the *k-th* lamina

<u>Quadratic failure criteria.</u>

For plane stress with $\sigma_3 = 0$, failure is predicted when

Tsai-Hill: $\dfrac{\sigma_1^2}{s_L^2} - \dfrac{\sigma_1\sigma_2}{s_L^2} + \dfrac{\sigma_2^2}{s_T^2} + \dfrac{\tau_{12}^2}{s_{LT}^2} \geq 1$

Tsai-Wu: $F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\tau_{12}^2 + F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 \geq 1$

where $F_{11} = \dfrac{1}{s_L^+ s_L^-}$, $F_{22} = \dfrac{1}{s_T^+ s_T^-}$, $F_1 = \dfrac{1}{s_L^+} - \dfrac{1}{s_L^-}$, $F_2 = \dfrac{1}{s_T^+} - \dfrac{1}{s_T^-}$, $F_{66} = \dfrac{1}{s_{LT}^2}$

F_{12} should ideally be optimised using appropriate strength data. In the absence of such data, a default value which should be used is

$$F_{12} = -\frac{\left(F_{11}F_{22}\right)^{1/2}}{2}$$

Fracture mechanics

Consider an orthotropic solid with principal material directions x_1 and x_2. Define two effective elastic moduli E'_A and E'_B as

$$\frac{1}{E'_A} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{22}}{S_{11}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

$$\frac{1}{E'_B} = \left(\frac{S_{11}S_{22}}{2}\right)^{1/2}\left(\left(\frac{S_{11}}{S_{22}}\right)^{1/2}\left(1+\frac{2S_{12}+S_{66}}{2\sqrt{S_{11}S_{22}}}\right)\right)^{1/2}$$

where S_{11} etc. are the compliances.

Then G and K are related for plane stress conditions by:

crack running in x_1 direction: $G_I E'_A = K_I^2 ; G_{II} E'_B = K_{II}^2$

crack running in x_2 direction: $G_I E'_B = K_I^2 ; G_{II} E'_A = K_{II}^2$.

For mixed mode problems, the total strain energy release rate G is given by

$G = G_I + G_{II}$

Approximate design data

	Steel	Aluminium	CFRP	GFRP	Kevlar
Cost C (£/kg)	1	2	100	5	25
E_1 (GPa)	210	70	140	45	80
G (GPa)	80	26	≈35	≈11	≈20
ρ (kg/m^3)	7800	2700	1500	1900	1400
e^+ (%)	0.1-0.8	0.1-0.8	0.4	0.3	0.5
e^- (%)	0.1-0.8	0.1-0.8	0.5	0.7	0.1
e_{LT} (%)	0.15-1	0.15-1	0.5	0.5	0.3

Table 1. Material data for preliminary or conceptual design. Costs are very approximate.

	Aluminium	Carbon/epoxy (AS/3501)	Kevlar/epoxy (Kevlar 49/934)	E-glass/epoxy (Scotchply/1002)
Cost (£/kg)	2	100	25	5
Density (kg/m^3)	2700	1500	1400	1900
E_1 (GPa)	70	138	76	39
E_2 (GPa)	70	9.0	5.5	8.3
ν_{12}	0.33	0.3	0.34	0.26
G_{12} (GPa)	26	6.9	2.3	4.1
s_L^+ (MPa)	300 (yield)	1448	1379	1103
s_L^- (MPa)	300	1172	276	621
s_T^+ (MPa)	300	48.3	27.6	27.6
s_T^- (MPa)	300	248	64.8	138
s_{LT} (MPa)	300	62.1	60.0	82.7

Table 2. Material data for detailed design calculations. Costs are very approximate.

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Engineering Tripos Part IIB: Module 4C2
Designing with Composites

Numerical answers - 2018/19

1. (b) $[Q] = \begin{bmatrix} 141.1 & 2.22 & 0 \\ 2.22 & 7.40 & 0 \\ 0 & 0 & 6.92 \end{bmatrix}$ GPa

1. (c) (i) $[A] = \begin{bmatrix} 716.5 & 201.3 & 0 \\ 201.3 & 716.5 & 0 \\ 0 & 0 & 257.6 \end{bmatrix}$ MNm^{-1}, $[B] = 0$

1. (c) (ii) $\varepsilon_x = \varepsilon_y = 0.000545$, $\gamma_{xy} = 0$

3. (b) (ii) $28° < \phi < 54°$.