Version HEMH/4

EGT3
ENGINEERING TRIPOS PART IIB

Tuesday 28 April 2015 2 to 3.30

Module 4C6

ADVANCED LINEAR VIBRATIONS

Answer not more than ***three*** *questions.*

All questions carry the same number of marks.

The ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

Write your candidate number ***not*** *your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (9 pages)
Supplementary pages: one extra copy of the chart P7 of the data sheet (Question 2) and one extra copy of Fig 2 (Question 3).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An impulse hammer is used to carry out a modal analysis of a solar panel for a satellite. Measurements are summarized in Table 1 for the point receptance near the middle of the panel. The mass of the panel is around 1kg and the perimeter of the panel can be assumed to be free.

Mode	Frequency (Hz)	Half-power bandwidth (Hz)	Modal amplitude (ms^{-1}/N)
1	0.2	0.02	0.8
2	0.6	0.03	0.25
3	0.65	0.0013	0.25
4	1.2	0.012	0.1

Table 1

(a) Estimate the Q-factor for each mode and sketch the transfer function with a dB vertical axis, taking care to show the form of the transfer function for frequencies close to zero. Use your sketch to explain what is meant by modal overlap. [40%]

(b) Sketch on a single diagram modal circles for each of the four modes. [30%]

(c) Discuss (with sketches) how the entries in Table 1 might be different if the measurements were made at a different point. Consider the possibility that measurements are made near a nodal line. [30%]

2 (a) Explain briefly how added damping layers can be used to control the damping of plate structures. Distinguish between the cases of *free-layer* and *constrained-layer* treatments. List the advantages and disadvantages of both types of treatment. [30%]

(b) The steel floor panel of a vehicle is to be modified with a free layer damping treatment. Using the correspondence principle together with information given in the data sheet, show that the Q factor of modes of the treated panel is given approximately by

$$Q = \frac{1}{eh\eta[4h^2 + 6h + 3]}$$

provided that the additional layer does not modify the mechanical properties of the original plate very greatly. The notation of the Data Sheet is used here, and the loss factor of the damping layer is denoted $\boldsymbol{\eta}$. State any assumptions that you make. [30%]

(c) To choose a suitable material for the damping layer, the data in the chart on page 7 of the Data Sheet can be used. Explain how the damping performance, using the approximate result of section (b), can be represented graphically in this chart. Give an example of two dissimilar materials that would result in similar Q factors for the treated plate, with a damping layer of given thickness. Discuss suitable materials to use for the damping treatment, explaining your reasoning and mentioning possible advantages and disadvantages. [40%]

3 Sound waves in a long duct of constant cross-section obey the governing equation

$$\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}+\frac{\partial^2 p}{\partial z^2}=\frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}$$

where $p(x,y,z,t)$ is the acoustical pressure associated with the sound wave, c is the speed of sound in air and t is time. Cartesian coordinates (x,y,z) are used, with the z-axis aligned with the axis of the duct. The appropriate boundary condition for a duct with hard walls is that the normal gradient of pressure is zero on the wall.

(a) Suppose the duct has circular cross-section, with radius a. For a sinusoidal wave propagating along the length of the duct, the pressure can be written in the form

$$p = f(x,y)e^{i(\omega t-\lambda z)}$$

where f is a function describing the variation of pressure in cross-sectional planes. Show that f obeys a very similar governing equation to that for the transverse vibration of a stretched circular membrane. What is the appropriate boundary condition on f? [25%]

(b) Using information from the Data Sheet, write down possible forms of the function f expressed in polar coordinates. Show that each "mode" of the corresponding membrane problem gives rise to a different type of travelling wave in the duct. What is the relation between frequency ω and wavenumber κ along the duct, for a given type of propagating wave? Show that the corresponding wave in the duct can only propagate for frequencies above a certain value. [30%]

(c) Using the graphs of the first few Bessel functions given in Fig. 1, estimate the values of this limiting frequency for the first three "modes" of the corresponding membrane problem, for a duct of diameter 1 m and assuming that c = 340 m/s. With the aid of sketches, describe the nature of the associated travelling waves. Estimate how many types of wave can propagate at 500 Hz. [30%]

(d) Find an expression for the speed of a propagating wave, and plot it as a function of frequency. [15%]

(cont.

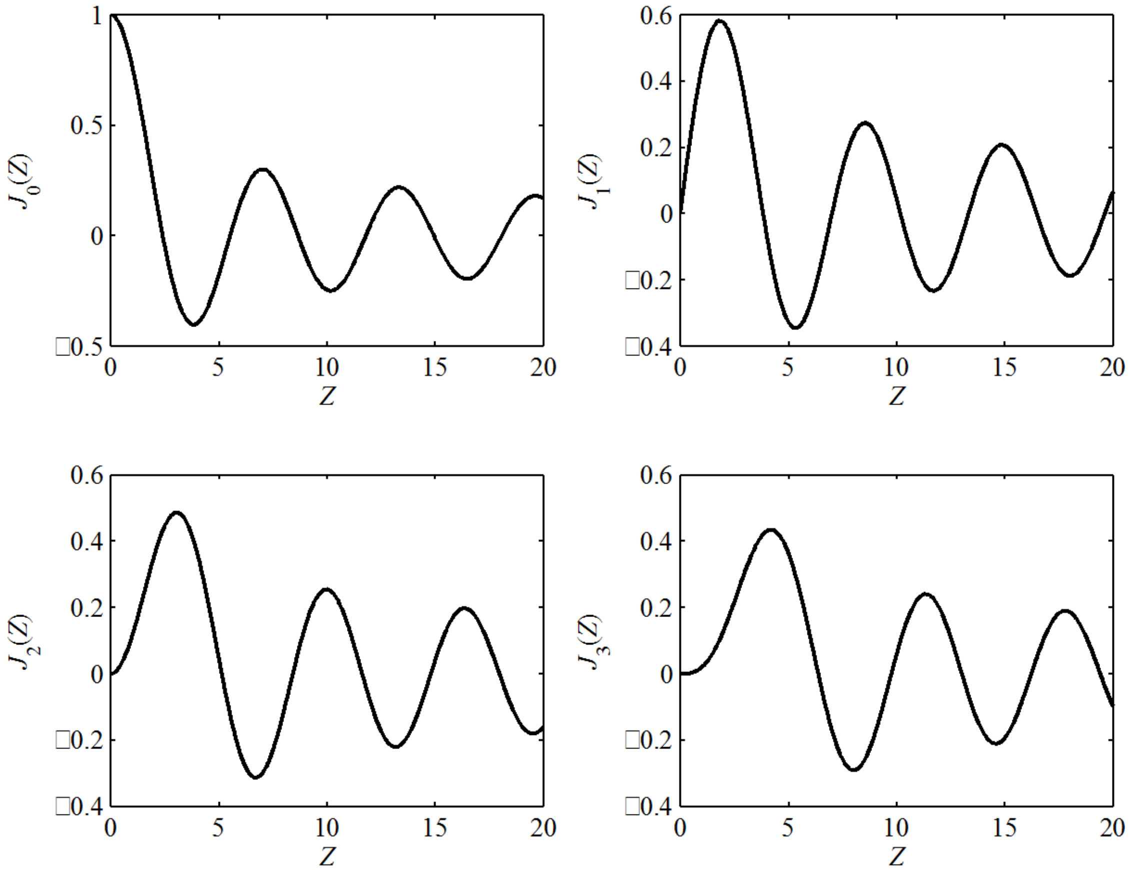

Fig. 1

4 (a) Two beams each have length L, bending rigidity EI and density ρ. One beam has free-free boundary conditions, the other pinned-free boundary conditions. Justify the use of the interlacing theorem to relate the natural frequencies of the two beams for small-amplitude bending vibration. [10%]

Given that the natural frequencies ω are determined by the roots of the equations

$$\cos \alpha L \cosh \alpha L = 1 \quad \text{(free-free)}$$

and

$$\tan \alpha L = \tanh \alpha L \quad \text{(pinned-free)}$$

where $\alpha^4 = \dfrac{\rho A}{EI}\omega^2$, give a graphical demonstration of this interlacing behaviour. [25%]

(b) Two parts of a car body panel are held together by a series of spot welds. One of the welds comes apart. What (if anything) does the interlacing theorem say about the effect on the natural frequencies of the panel if the spot weld is treated as (i) a pinned constraint; or (ii) a clamped constraint? [25%]

(c) The body of an acoustic guitar consists of flexible panels forming an enclosure which connects to the surrounding air via a soundhole. Six tensioned strings are fitted to the guitar. Discuss what can be deduced from the interlacing theorem about the effect on the natural frequencies of:

(i) gluing a point mass to the bridge of the guitar; [10%]

(ii) blocking the soundhole with a rigid plug (without changing the mechanical behaviour of the surrounding plates); [10%]

(iii) using a finger to change the note on one string by holding it against a fret and thus shortening the vibrating length. [20%]

END OF PAPER