EGT3
ENGINEERING TRIPOS PART IIB

Monday 25 April 2016 9.30 to 11.00

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (9 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An impulse hammer of mass 0.1 kg is to be used to conduct a modal test on a uniform cantilever beam. The beam is clamped rigidly at one end as shown in Fig. 1 and an accelerometer is fixed to the other end of the beam at A, near one corner. During the test both flexural and torsional modes are to be excited by the hammer. The first three flexural modes are thought to have frequencies around 20 Hz, 130 Hz and 350 Hz and the frequencies of the first three torsional modes are thought to be around 100 Hz, 300 Hz and 500 Hz.

(a) (i) Explain clearly why a hammer pulse of 1ms duration will be suitable for investigating these six modes of vibration. What are the disadvantages of a shorter or longer pulse duration? [15%]

(ii) What value for the stiffness of the hammer tip will give a suitable pulse duration? [5%]

(b) The force transducer in the hammer has a sensitivity of 5 pC/N and the hammer strikes the beam with a velocity of around 3 m s^{-1} . Design a simple charge amplifier to produce an output signal suitable for a data logger with an input range of $\pm$ 5 V. [20%]

(c) Select a suitable sampling rate for the data logger so that the use of an antialiasing filter is not required. [20%]

(d) Without detailed calculation *sketch* the mode shapes for

(i) the three flexural modes; [10%]

(ii) the three torsional modes. [10%]

(e) Without detailed calculation *sketch* the transfer function that might be measured between

(i) points A and B; [10%]

(ii) points A and C. [10%]

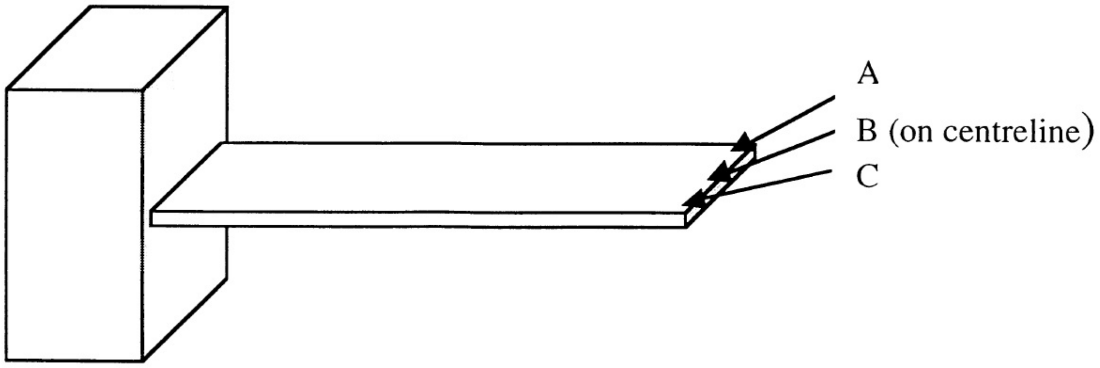

Fig. 1

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2 A “bass reflex” loudspeaker can be idealised as shown in Fig. 2. A piston of mass M and area A (the loudspeaker cone) is supported by a spring of stiffness K, and is enclosed by a rigid box of volume V which has a single opening in the form of a neck of area S and effective length L (including any end corrections).

(a) Explain briefly why the air in the neck region can be treated approximately as a rigid mass. [10%]

(b) If this mass has outward displacement Y while the loudspeaker cone has outward displacement X, show that the system can be represented as an equivalent mass-spring system as shown in Fig. 3 with the definitions $x = AX$ and $y = -SY$, in terms of notation shown in Fig. 3. Show that

$$k_b = \frac{c^2\rho}{V}$$

and find expressions for k_a, m_a and m_b. You may assume that pressure perturbations p' in the box are related to density perturbations ρ' by $p' = c^2\rho'$. [30%]

(c) Find the natural frequencies and describe the mode shapes for the special cases when:

(i) the opening is blocked; [15%]

(ii) the loudspeaker cone is immobilised; [15%]

(iii) $K = 0$. [20%]

For case (ii), verify that the standard result for a Helmholtz resonator is recovered. In all cases, comment on whether the interlacing theorem implies any relationship to the frequencies in the general case.

(d) Comment briefly on the possible advantages of the bass-reflex design over the *infinite baffle* design from section (c)(i). [10%]

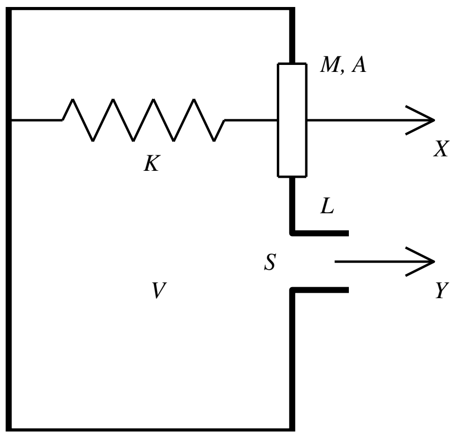

Fig. 2

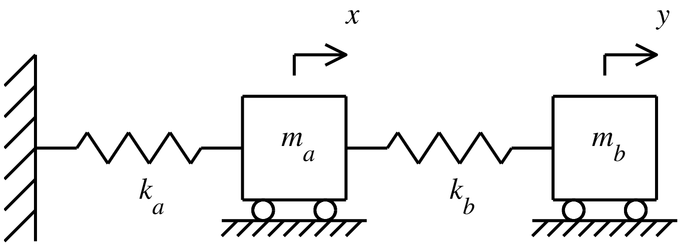

Fig. 3

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3 (a) The mass-spring system illustrated in Fig. 4 consists of three masses connected by springs and one dashpot, with values as labelled in the figure. For the case $c = 0$, taking advantage of symmetry and orthogonality or otherwise, write down the mode shapes and hence obtain expressions for the natural frequencies. [30%]

(b) For the case with $c \neq 0$, find the mass, stiffness and dissipation matrices and hence find the matrix A based on the first-order method as defined in the Data Sheet. For a particular system having $m = 1$ kg, $k = 1000$ N/m and $c = 3$ N s/m, the Matlab function call `[V,D]=eig(A)` yields the result shown in Figure 5. Explain these results, and describe the three modes in relation to those of the undamped system from part (a). [40%]

(c) For each mode, sketch a phasor diagram. For the mode with the intermediate frequency, sketch how the modal circles might appear from a measurement of the velocity of each mass in response to excitation on the left-hand mass. Significant features should be labelled. [30%]

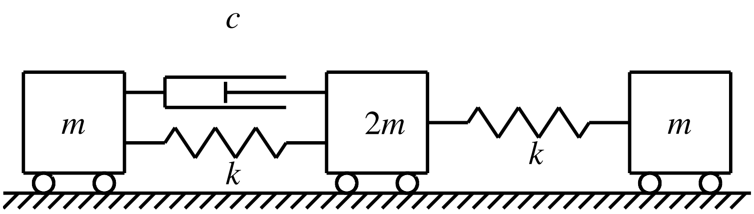

Fig. 4

```
V =
 -0.0004-0.0130i   -0.0004+0.0130i    0.0026+0.0221i    0.0026-0.0221i   -0.5774-0.0000i   -0.5774+0.0000i
  0.0021+0.0127i    0.0021-0.0127i   -0.0011+0.0001i   -0.0011-0.0001i   -0.5774-0.0000i   -0.5774+0.0000i
 -0.0038-0.0123i   -0.0038+0.0123i   -0.0005-0.0223i   -0.0005+0.0223i   -0.5774           -0.5774
  0.5815            0.5815           -0.7023+0.0672i   -0.7023-0.0672i   -0.0000-0.0000i   -0.0000+0.0000i
 -0.5686+0.0764i   -0.5686-0.0764i   -0.0024-0.0336i   -0.0024+0.0336i    0.0000-0.0000i    0.0000+0.0000i
  0.5557-0.1528i    0.5557+0.1528i    0.7072            0.7072           -0.0000-0.0000i   -0.0000+0.0000i


D =
 -1.5000+44.5949i        0                 0                 0                 0                 0
       0           -1.5000-44.5949i        0                 0                 0                 0
       0                 0           -0.7500+31.6856i        0                 0                 0
       0                 0                 0           -0.7500-31.6856i        0                 0
       0                 0                 0                 0           0.0000+0.0000i          0
       0                 0                 0                 0                 0           0.0000-0.0000i
```

Fig. 5

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4 (a) (i) A sandwich panel for use in a car roof consists of two sheets of aluminium glued together with polymer-based glue. Discuss the mechanism(s) of energy dissipation when the panel undergoes bending vibration. [15%]

(ii) A magnet is attached to a steel panel which undergoes bending vibration. Discuss the mechanism(s) of energy dissipation associated with the magnet. [15%]

(iii) The high-pressure turbine fans in jet engines often make use of *under-platform dampers*, metal rods in contact with the fan blades in such a way that blade vibration induces frictional energy loss. Discuss why this particular choice of damping method is chosen in that application. [15%]

(b) (i) A two-degree-of-freedom vibrating system is illustrated in Fig. 6, with masses and stiffnesses as labelled there. For the undamped case with $\eta = 0$, find expressions for the natural frequencies and mode shapes of this system. [30%]

(ii) For the damped case with $\eta > 0$, use Rayleigh's principle to estimate the Q factors of each mode of vibration, on the assumption that η is small. [25%]

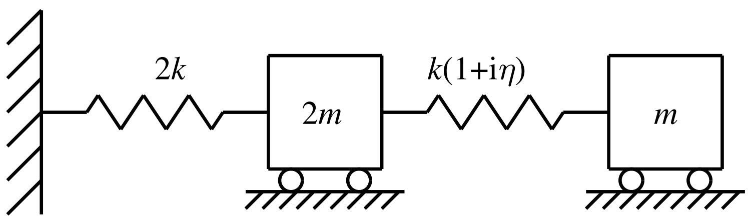

Fig. 6

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Answers: 4C6 2016, Assessor Dr T Butlin

1) Note: values are approximate due to rounding for this question
   a)
      i) /
      ii) k=900kN/m
   b) C = 1nF
   c) fs = 10kHz

2)
   a) /
   b) $m_a = {}^{M}/_{A^2}$; $m_b = {}^{\rho L}/_{S}$; $k_a = {}^{K}/_{A^2}$; $k_b = {}^{c^2\rho}/_{V}$
   c)
      i) $\omega_{(i)}^2 = \frac{k_a+k_b}{m} = \frac{K+c^2A^2\rho/V}{M}$
      ii) $\omega_{(ii)}^2 = \frac{k_b}{m_b} = \frac{c^2S}{LV}$
      iii) $\omega_{(iii)}^2 = k_b\left(\frac{1}{m_a}+\frac{1}{m_b}\right)$

3)
   a) $\omega_0 = 0$; $u_1 = [1 \quad 1 \quad 1]^T$; $\omega_1 = \sqrt{\frac{k}{m}}$; $u_1 = [1 \quad 0 \quad -1]^T$; $\omega_2 = \sqrt{\frac{2k}{m}}$; $u_2 = [1 \quad -1 \quad 1]^T$
   b) $$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{k}{m} & \frac{k}{m} & 0 & -\frac{c}{m} & \frac{c}{m} & 0 \\ \frac{k}{2m} & -\frac{k}{m} & \frac{k}{2m} & \frac{c}{2m} & -\frac{c}{2m} & 0 \\ 0 & \frac{k}{m} & -\frac{k}{m} & 0 & 0 & 0 \end{bmatrix}$$
   c) /

4)
   a) /
   b)
      i) $\omega_1^2 = \frac{k}{2m}$; $u_1 = [1 \quad 2]$; $\omega_2^2 = \frac{2k}{m}$; $u_2 = [1 \quad -1]$
      ii) $Q_1 = \frac{3}{\eta}$; $Q_2 = \frac{3}{2\eta}$