Version HEMH/2

EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 1 May 2019 2 to 3.40

Module 4C6

ADVANCED LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C6 Advanced Linear Vibration data sheet (9 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An instrumented impulse hammer is to be used for modal testing. The total mass of the hammer head is m and the stiffness of the hammer tip is k . The hammer produces a half-sinusoidal force pulse of duration b and the peak force is F_0.

(a) (i) Sketch a typical instrumented hammer and identify its principal parts. [10%]

(ii) Find an expression for b in terms of k and m. [10%]

(iii) Sketch the impulsive force $f(t)$ and the impulse spectrum $F(\omega)$, showing salient values. With reference to your sketches explain the "rule of thumb" that an impulse does not excite vibration at frequencies (in Hz) above 1/b. [20%]

(b) The hammer is fitted with a piezo force transducer which generates a charge output $Q(t)$. A charge amplifier is used to amplify $Q(t)$ and to generate an output voltage $V(t)$.

(i) Sketch a simple OpAmp circuit for this purpose. [10%]

(ii) Modify your circuit to incorporate a high-pass filter and explain why this is necessary. [10%]

(iii) Explain why it might be necessary to include a low-pass filter. What measures within the hammer might be taken to obviate the need for a low-pass filter? [20%]

(iv) Sketch the form of $V(t)$ and identify key features that do not appear on your sketch of $f(t)$ in (a)(iii). [20%]

2 The differential equation of motion of a rectangular membrane that has an out-of-plane displacement $w(x_1, x_2, t)$ is given by

$$T_1 \frac{\partial^2 w}{\partial x_1^2} + T_2 \frac{\partial^2 w}{\partial x_2^2} - m \frac{\partial^2 w}{\partial t^2} = 0$$

where T_1 and T_2 are the in-plane tensions acting respectively in the x_1 and the x_2 directions and m is the mass per unit area. The membrane covers the area $0 \leq x_1 \leq L_1$, $0 \leq x_2 \leq L_2$ and all the boundaries of the membrane are restrained from motion.

(a) By assuming a separable solution to the equation of motion, show that the mode shapes of the membrane must have the form

$$\sin(k_1 x_1) \sin(k_2 x_2)$$

for appropriate values of the constants k_1 and k_2. [30%]

(b) Proceed to find an expression for the natural frequencies of the membrane. [20%]

(c) The membrane is now considered to have free edges rather than constrained edges, so that

$$\frac{\partial w}{\partial x_1} = 0 \text{ on } x_1 = 0 \text{ and } x_1 = L_1, \text{ and } \frac{\partial w}{\partial x_2} = 0 \text{ on } x_2 = 0 \text{ and } x_2 = L_2.$$

Calculate the new natural frequencies and modes shapes. [40%]

(d) If each of the two membranes is struck with a drum stick, would you be able to hear the difference? [10%]

3 A beam of rectangular cross section has elastic modulus E and mass per unit length m. The length of the beam is L, its width is b and its depth is $2d$. A damping layer of thickness t is applied to the top surface of the beam over the full width of the beam, but restricted to the region $\alpha \leq x \leq \beta$, where the coordinate x measures the distance along the neutral axis of the beam, so that the ends of the beam lie at $x = 0$ and $x = L$. The damping treatment is intended to damp the motion of the beam in the first mode, for which the deflection of the neutral axis is

$$w(x) = w_0 \sin(\pi x/L)$$

(a) Assuming that the mode shape is unchanged by the addition of the damping treatment, show that axial strain in the damping treatment can be approximated as $\epsilon(x) = d(\partial^2 w/\partial x^2)$. Hence show that the strain energy of the damping treatment is given by

$$U_a = (E_a b t d^2/2) \int_{\alpha}^{\beta} \left(\partial^2 w/\partial x^2\right)^2 dx$$

where E_a is the elastic modulus of the damping treatment. [25%]

(b) If the damping treatment is now taken to have a complex modulus in the form $E_a(1 + i\eta_a)$, use the correspondence principle in conjunction with the Rayleigh quotient to obtain the loss factor for the beam (assume that the mass of the damping treatment can be neglected). [40%]

(c) The bending rigidity of a 2-layer beam is given on page 8 of the 4C6 data sheet. Compare your answer in part (b) to that which would arise from using the data sheet, for the case in which the damping treatment covers the whole of the top surface of the beam. Show that the two answers agree providing $t \ll d$. [35%]

4 A piston of mass M, thickness d, and radius R is free to slide in a cylinder, as shown in Fig, 1. The piston is mounted on a spring of stiffness K, and in the equilibrium position the piston is located at a distance D from the base of the cylinder. The piston has a circular hole of radius $a \ll R$, and the air density is ρ . Gravity can be ignored.

(a) By representing the system as a two degree of freedom system, derive the governing equations of motion. Note that the change in pressure Δp in the cylinder can be related to the change in density $\Delta\rho$ via the formula $\Delta p = c^2\Delta\rho$ where c is the speed of sound. [30%]

(b) Calculate the natural frequency of the system for the case in which the spring is very stiff, so that the piston is restrained from moving. [10%]

(c) Calculate the natural frequency of the system for the case in which the hole in the cylinder is blocked, so that air flow is prevented. [10%]

(d) Calculate the coupled natural frequencies of the system for the case $K = 0$. [30%]

(e) Demonstrate that your answers to parts (b) to (d) are consistent with the interlacing theorem. [10%]

(f) How would the natural frequencies change were the hole located in the base of the cylinder rather than in the piston? [10%]

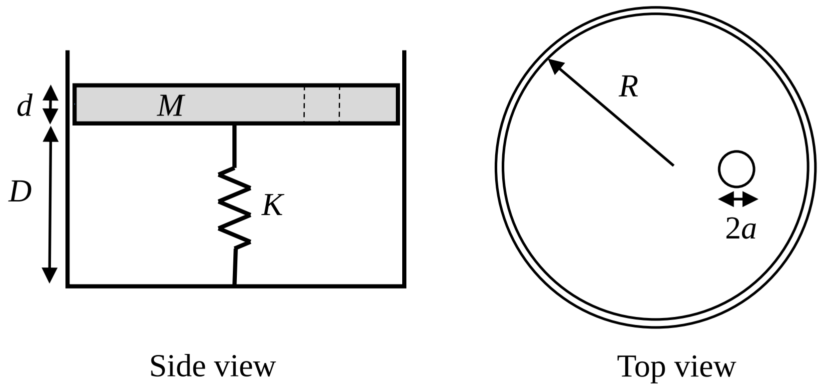

Fig. 1

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