Version RSL/3

EGT3
ENGINEERING TRIPOS PART IIB

Thursday 30 April 2015 9.30 to 11.00

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C7 Random and Non-linear Vibrations data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 An electro-mechanical energy harvesting device consists of a damped mass/spring system which is connected to a capacitor and a resistor. The device is mounted on a vibrating surface, and the power dissipated in the resistor can be considered to be harvested (the resistor is a representation of a harvesting circuit). The governing equations are

$$m\ddot{x} + b\dot{x} + kx + \lambda v = ma \ ,$$

$$C\dot{v} + v / R = \lambda \dot{x} \, ,$$

where m, b, and k are respectively the mass, damping, and stiffness of the device, λ is an electro-mechanical coupling coefficient, C and R are the capacitance and resistance of the electrical components, x is the displacement of the device relative to the vibrating surface to which it is attached, v is the generated voltage, and a is the acceleration of the vibrating surface. The acceleration of the vibrating surface is random white noise with double-sided spectral density S_0.

(a) Find an expression for the transfer function between the displacement of the device x and the acceleration of the surface a. Hence find an expression for the spectrum of the *velocity* $\dot{x}$ of the device. [30%]

(b) For the case in which the capacitance is zero, C=0, use the usual white noise formulae to derive expressions for the mean squared displacement and velocity of the device. Find the average power dissipated by the resistor, and by the damper, and show that the sum of these two quantities depends only on the mass of the device and S_0. [30%]

(c) The device will fail due to excessive stresses if the displacement exceeds five times the root mean squared displacement. It is observed that the response is very narrow banded, and that during a one hour period there are approximately 1000 crossings of zero with positive velocity. Calculate the probability that the device will fail during a *three* hour period for the case C=0. [30%]

(d) If the *velocity* of the vibrating surface, rather than the acceleration, is approximated as white noise, show from your answer to part (a) above that the predicted mean squared velocity of the system is not physically reasonable regardless of the value of C. [10%]

2 A floating offshore platform is moored by anchor cables that have a non-linear force-displacement relationship. It is found that during a severe storm, the joint probability density function of the displacement $x(t)$ and velocity $\dot{x}(t)$ of the vessel has the form

$$p(x, \dot{x}) = C \exp\{-(\dot{x}/a)^2 - (x/d)^4\},$$

where C, a, and d are constants.

(a) Derive an expression for the mean rate at which the vessel crosses a response level $x=b$ with positive velocity. [25%]

(b) Assuming that the response is narrow-banded, derive an expression for the probability density function of the peaks of the response. [25%]

(c) The fatigue failure of an anchor cable under repeated loading follows the law

$$N(S) = \alpha S^{-2},$$

where $N(S)$ is the number of cycles to failure under stress cycles of amplitude S, and α is a constant. Although the anchor cables produce a non-linear force-displacement relationship, the stress in an anchor cable is approximately proportional to the vessel displacement so that the stress produced by the vessel displacement is βx. Show that the average fatigue damage that is caused by a cycle of the vessel response is

$$\mathrm{E}[1/N(S)] = \frac{\beta^2 d^2 \sqrt{\pi}}{2\alpha}.$$ [25%]

(d) Derive an expression for the fatigue life (i.e. the time to failure) of the anchor cable under the storm conditions. [25%]

 (TURN OVER

3 A nonlinear undamped single degree-of-freedom vibratory system of mass *m* has a force-displacement characteristic comprising a symmetrical dead-zone of width 2*a* as shown in Figure 1.

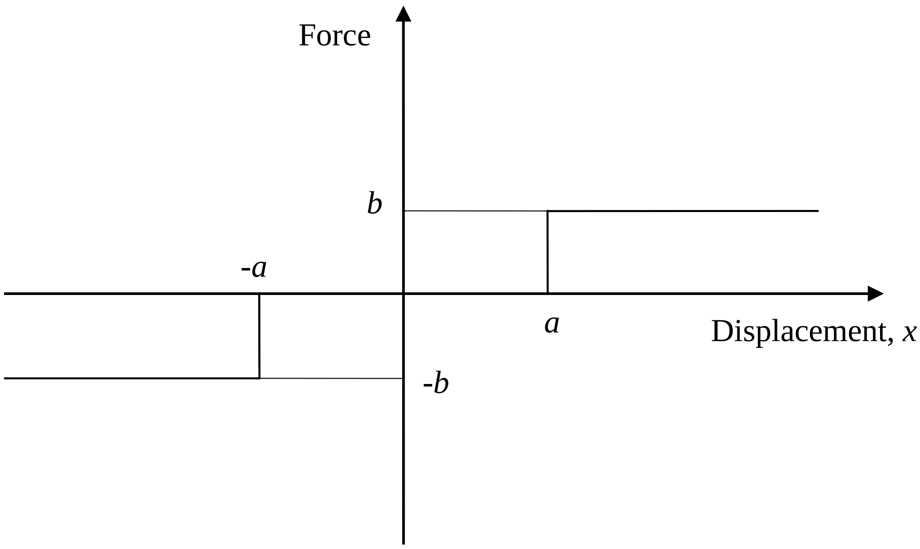

Figure 1

(a) Sketch the input and output waveforms when the system is sinusoidally driven with amplitude $A > a$. [20%]

(b) Determine the Describing Function for a sinusoidal input of amplitude *A*. [50%]

(c) If the system is driven by a force $f \cos \omega t$ determine an approximate relation between the response amplitude and *A* and ω. [30%]

4 A non-linear system is described by the set of equations:

$$\dot{x} = y - y^3,$$
$$\dot{y} = -\alpha x - y^2,$$

where α is a real-valued constant.

(a) Determine the singular points for the system. [20%]

(b) Determine the type and stability of each singular point. [40%]

(c) Sketch the behaviour of the system in the phase plane when $\alpha = 1$. [40%]

END OF PAPER

THIS PAGE IS BLANK

ANSWERS

1 (a) $H(\omega) = m[-m\omega^2 + i\omega b + k + \lambda^2 i\omega / (Ci\omega + 1/R)]^{-1}$, $S_{\dot{x}\dot{x}}(\omega) = \omega^2 \left|H(\omega)\right|^2 S_0$.

(b) $\sigma_x^2 = \pi S_0 m^2 / [k(b + \lambda^2 R)]$, $\sigma_{\dot{x}}^2 = \pi S_0 m / (b + \lambda^2 R)$, $P = \pi S_0 m$.

(c) $P_{\text{fail}} = 0.011$.

2 (a) $\nu_b^+ = (1/2)Ca^2 \exp[-(b/d)^4]$.

(b) $p(b) = (4b^3 / d^4)\exp[-(b/d)^4]$.

(d) $T = 4\alpha / (\sqrt{\pi} C\beta^2 d^2 a^2)$.

3 (b) $DF = (4b / A\pi)\sqrt{1-(a/A)^2}$ for $A > a$; $DF = 0$ for $A < a$.

(c) $f = -\omega^2 mA + (4b/\pi)\sqrt{1-(a/A)^2}$ for $A > a$; $f = -\omega^2 mA$ for $A < a$.

4 (a) $(0,0)$, $(-1/\alpha, 1)$, $(-1/\alpha, -1)$.

(b) First point: $\alpha < 0$ saddle point, $\alpha > 0$ centre

Second point: $\alpha < -0.5$ stable focus, $-0.5 < \alpha < 0$ stable node, $\alpha > 0$ saddle point

Third point: $\alpha < -0.5$ unstable focus, $-0.5 < \alpha < 0$ unstable node, $\alpha > 0$ saddle point