EGT3
ENGINEERING TRIPOS PART IIB

Wednesday 26 April 2017 2 to 3.30

Module 4C7

RANDOM AND NON-LINEAR VIBRATIONS

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4C7 Random and Non-linear Vibrations data sheet (4 pages).
Engineering Data Book

10 minutes reading time is allowed for this paper.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

1 Non-linear systems that are subjected to random excitation are often analysed by replacing non-linear terms with linear terms in such a way as to minimise the resulting error. For example, a cubic stiffness term can be written as

$$kx^3 = k_e x + \varepsilon \ ,$$

where k is the non-linear stiffness coefficient, k_e is the "best fit" equivalent linear stiffness, and ε is an error term that is neglected in the analysis.

(a) Show that if x is a Gaussian random process then the value of k_e that minimises the mean squared value of the error in the above equation is

$$k_e = 3k\sigma_x^2,$$

where σ_x is the rms value of x (note the hint at the end of the question). [20%]

(b) A non-linear oscillator has the equation of motion

$$m\ddot{x} + c\dot{x} + k_1 x + k_3 x^3 = F(t),$$

where m, c, k_1, and k_3 are the relevant system parameters, and $F(t)$ is a white noise excitation with double-sided spectral density S_0. By replacing the non-linear stiffness by a linear stiffness (and neglecting the error term) derive an expression for the rms response of the system σ_x. [30%]

(c) Derive an expression for the mean rate of power dissipated by the damping term in the system in part (b). [10%]

(d) An oscillator with non-linear damping has the equation of motion

$$m\ddot{x} + c\dot{x}^5 + kx = F(t),$$

where m, c, and k are the relevant system properties and $F(t)$ is again white noise excitation with double-sided spectral density S_0. By using the linearization technique, derive an expression for the rms velocity of the system and the mean rate of power dissipation. [40%]

Hint: if y is a Gaussian random process then $\mathrm{E}[y^4] = 3\sigma_y^4$ and $\mathrm{E}[y^6] = 15\sigma_y^6$

2 The two-degree-of-freedom system shown in Fig. 1 represents a simplified model of two circuit boards in an equipment rack in a satellite. There is concern that the boards will impact with each other during launch causing damage. The two boards are of mass m_1 and m_2, the springs shown in the figure are each of stiffness k and the coefficient of the damper is c. The degrees of freedom x_1 and x_2 represent the dynamic displacements of the boards away from their equilibrium positions, and in the absence of vibration the separation of the masses is b. Mass m_1 is subjected to a random force $F(t)$ that has double sided spectral density $S_{FF}(\omega)$.

(a) Derive an expression for the spectrum of the relative displacement $y = x_2 - x_1$ between the masses. Explain in physical terms why the spectrum is zero if $m_2 = 0$. [50%]

(b) During a vibration test it is found that

$$\mathrm{E}[x_1^2] = \mathrm{E}[x_2^2] = 1\ \mathrm{mm}^2, \qquad \mathrm{E}[\dot{x}_1^2] = \mathrm{E}[\dot{x}_2^2] = 4\times10^5\ \mathrm{mm}^2\mathrm{s}^{-2},$$

and that the correlation coefficient ρ between x_1 and x_2 has the same value as the correlation coefficient between $\dot{x}_1$ and $\dot{x}_2$. For the case $b = 6$ mm and $\rho = -0.5$ calculate the probability that the two boards will impact at least once if the excitation lasts for 30 s. Repeat this calculation for the case $b = 6$ mm and $\rho = 0.5$, and explain in physical terms the change in the computed probability. [50%]

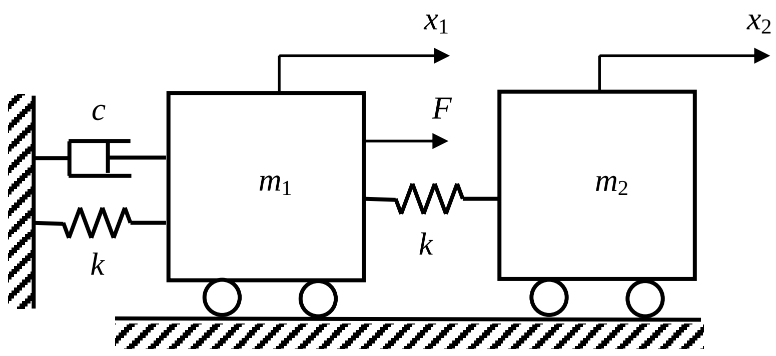

Fig. 1

3 Consider the conservative non-linear system for a particle of unit mass with dynamics specified by the equation

$$\ddot{x} = x - x^2 .$$

(a) Find the singular points and establish their type. [20%]

(b) Sketch the phase portrait for the system. [40%]

(c) Write an equation for the potential energy as a function of the displacement *x*. [10%]

(d) Find an equation for the trajectories that pass through the saddle point. [30%]

4 A non-linear oscillator is excited by two harmonic forcing functions with distinct frequencies, Ω_1 and Ω_2, with the dynamics represented by the equation

$$\ddot{x} + p^2 x + \varepsilon x^2 = F_1 \cos \Omega_1 t + F_2 \cos \Omega_2 t .$$

(a) Obtain a solution for the linear problem by setting ε=0. [10%]

(b) Use the method of iteration to solve for the response x_1 to first order in ε. [70%]

(c) Identify the combination harmonics that arise in the response due to the operative non-linearity. How may these results be practically employed for the excitation of a non-linear system represented by the above dynamics? [20%]

END OF PAPER